Archdiocese of Hartford

Mathematics Curriculum Standards

2012

Grades 1-12

Office of Catholic Schools
Archdiocese of Hartford
467 Bloomfield Avenue
Bloomfield, CT 06002
www.catholicschoolshartford.org

"Mathematics may not teach us how to inhale oxygen and exhale carbon dioxide, or to love a friend and forgive an enemy. But it gives us every reason to hope that every problem has a solution."

Catholic Schools
Education for a Lifetime

Purpose and Vision for Catholic School Education

Catholic Schools in the Archdiocese of Hartford welcome students of all faiths, ethnic groups and socio-economic backgrounds. The fundamental purpose of Catholic schools is to:

Provide a safe, nurturing and secure environment in which **students encounter the living God**, who in Jesus Christ, reveals His transforming love and truth;

Partner with parents to support students in their learning and in their **search for knowledge, meaning, and truth**;

Create a Catholic climate that contributes to the **formation of students** as active participants in the parish community;

Foster a **culture of educational excellence** through critical thinking skills, innovative rigorous curriculum standards, a global perspective, and an emphasis on moral education, community, and service;

Promote life-long learning that advances the **development of the whole person** - mind, body, and soul; and

Graduate students prepared to become **productive, virtuous citizens and church leaders** who will fashion a more humane and just world.

OFFICE OF CATHOLIC SCHOOLS
ARCHDIOCESE OF HARTFORD
467 BLOOMFIELD AVENUE • BLOOMFIELD, CONNECTICUT 06002-2999
TEL. (860) 242-4362 • FAX (860) 242-8683
www.catholicschoolshartford.org

Office of the
Superintendent
of Catholic Schools

Fall 2012

Dear Colleagues in Catholic School Education:

Peace and greetings!

It is with pride and pleasure that I approve the revised *Mathematics Curriculum Standards 2012 Merged with the Common Core State Standards* for grades 1-12. The study of mathematics is one of the 21st century's most significant challenges. The study of math and its advanced courses is vital to successful high school and college experiences. Successful participation in a fast-paced, far-reaching, borderless world demands excellent training in mathematics. From concrete counting to the abstract use of formulas, mathematics gives meaning to our world, and hope for future generations.

> *Today, perhaps more than ever it is important to recognize that learning is a lifelong experience. Rapid, radical changes in contemporary society demand well planned, continuing efforts to assimilate new data, new insights, new modes of thinking and acting.* *(To Teach as Jesus Did, 1972, #43)*

You, the educators, must understand that well-honed skills, creative use of technology, and creative problem solving skills are vital to a successful program of mathematics' study. The use of data will be used for life altering decisions. It is an awesome responsibility of educators to appropriately instruct our students in the application of mathematics as well as the morality implicit in that decision making.

I am grateful to Mrs. Valerie Mara, Director of Curriculum Design, and her committee who so carefully and responsibly have addressed the merged standards, so that our students will have the finest tools and most current training in mathematics in order to make the world a better place for all who will follow.

Yours in Christ Jesus,

Dale R. Hoyt
Superintendent of Catholic Schools
Archdiocese of Hartford

Catholic Schools - Education for a Lifetime

Rationale for Learning Mathematics in the

Catholic Schools of the Archdiocese of Hartford

The vision statement of the Catholic schools of the Archdiocese of Hartford challenges schools to foster a culture of excellence through critical thinking, innovative and rigorous curriculum standards, a global perspective, and an emphasis on moral education, community and service. The discipline of mathematics is key to the achievement of that vision. The study of mathematics is the study of relationships, structure, and problem solving. Through math, students learn about patterns, chance, form, algorithms and change. They learn to observe, predict, analyze, and solve problems related to routine daily tasks.

Students learn to be creative and collaborative in problem solving. Significant moral decisions require the techniques of problem solving learned in a strong mathematics curriculum. By its nature, mathematics promotes logical and abstract thinking. The methodical approach needed to reach conclusions fosters the self-discipline necessary to solve simple and complex exercises. Knowledge of mathematical processes and skills are the tools needed to solve problems and construct valid arguments in other disciplines. Mathematics serves as a tool in both the natural and social sciences and stands as a logical foundation for the consideration of moral and ethical issues by Catholic Christian thinkers.

Modern technology requires varying forms of mathematical thought from all who use and create it. Mathematics can be appreciated in its purest form as an abstract art with order and pattern serving to reveal the beauty of God's creation. Most significant of all, the importance of mathematics instruction and learning lies in the universality of its problem solving applications to everyday life.

Defining Characteristics of Catholic Schools

The following ***Defining Characteristics*** flow directly from the Holy See's teaching on Catholic schools as compiled by Archbishop J. Michael Miller, CSB (*The Holy See's Teaching on Catholic Schools*, 2006), and from statements by Pope Benedict XVI and the American bishops. The characteristics define the deep Catholic identity of Catholic schools and serve as the platform on which the standards and benchmarks rest. The defining characteristics authenticate the standards and benchmarks, justifying their existence and providing their meaning.[1] These characteristics provide the foundation for teaching and learning in all our schools. Purposeful and deliberate integration of these characteristics in all lesson planning defines us as Catholic schools in the Archdiocese of Hartford.

Defining Characteristics of Catholic Schools
1. Centered in the Person of Jesus Christ Catholic education is rooted in the conviction that Jesus Christ provides the most comprehensive and compelling example of the realization of full human potential. (*The Catholic School,* 34, 35) In every aspect of programs, life, and activities, Catholic schools should foster personal relationship with Jesus Christ and communal witness to the Gospel message of love of God and neighbor and service to the world, especially the poor and marginalized. (Miller, *The Holy See's Teachings on Catholic Schools*, 25–26)
2. Contributing to the Evangelizing Mission of the Church By reason of its educational activity, Catholic schools participate directly and in a privileged way in the evangelizing mission of the church. (*The Catholic School*, 9; *The Catholic School on the Threshold of the Third Millennium*, 5, 11; *The Religious Dimensions of Education in a Catholic School*, 33) As an ecclesial entity where faith, culture, and life are brought into harmony, the Catholic school should be a place of real and specified pastoral ministry in communion with the local Bishop. (*The Catholic School,* 44; *The Catholic School on the Threshold of the Third Millennium*, 14*; The Religious Dimension of Education in a Catholic School,* 34*;*) The environment in Catholic schools should express the signs of Catholic culture, physically, and visibly. (*The Religious Dimension of Education in a Catholic School*, 25; Miller, 40)
3. Distinguished by Excellence Church documents, history, and practices, supported by Canon Law, establish that first and foremost a Catholic school is characterized by excellence. Consistent with the defining characteristics, Catholic schools should implement on-going processes and structures and gather evidence to ensure excellence in every aspect of its programs, life, and activities. (*Gravissimum Educationis* 8 and 9; *Code of Canon Law,* Canon 806 #2)

[1] Center for Catholic School Effectiveness, School of Education, Loyola University Chicago in partnership with Roche Center for Catholic Education, Lynch School of Education, Boston College (2012)

4. Committed to Educate the Whole Child
Catholic school education is rooted in the conviction that human beings have a transcendent destiny, and that education for the whole person must form the spiritual, intellectual, physical, psychological, social, moral, aesthetic and religious capacities of each child. Catholic schools should develop and implement academic, co-curricular, faith-formation, and service/ministry programs to educate the whole child in all these dimensions. (*The Catholic School*, 29) *National Standards and Benchmarks for Effective Catholic Elementary and Secondary Schools* Center for Catholic School Effectiveness, School of Education, Loyola University Chicago in partnership with Roche Center for Catholic Education, Lynch School of Education, Boston College (2012) 6

5. Steeped in a Catholic Worldview
Catholic education aims at the integral formation of the human person, which includes "preparation for professional life, formation of ethical and social awareness, developing awareness of the transcendental, and religious education" (*The Catholic School*, 31). All curriculum and instruction in a Catholic school should foster: the desire to seek wisdom and truth, the preference for social justice, the discipline to become self-learners, the capacity to recognize ethical and moral grounding for behavior, and the responsibility to transform and enrich the world with Gospel values. The Catholic school should avoid the error that its distinctiveness rests solely on its religious education program. (Miller, 43–45, 52)

6. Sustained by Gospel Witness
Catholic schools pay attention to the vocation of teachers and their participation in the Church's evangelizing mission. (*The Catholic School on the Threshold of the Third Millennium,* 19; *Lay Catholics in Schools*, 37) A Catholic educator is a role model for students and gives testimony by his or her life and commitment to mission. (Benedict XVI, June, 2005; Miller, 53) As much as possible, Catholic schools should recruit teachers who are practicing Catholics, who can understand and accept the teachings of the Catholic Church and the moral demands of the Gospel, and who can contribute to the achievement of the school's Catholic identity and apostolic goals, including participation in the school's commitment to social justice and evangelization. (United States Conference of Catholic Bishops, *National Directory for Catechesis,* 231)

7. Shaped by Communion and Community
Catholic school education places an emphasis on the school as community—an educational community of persons and a genuine community of faith. (*Lay Catholics in Schools*, 41, 22) Catholic schools should do everything they can to promote genuine trust and collaboration among teachers, with parents as the primary educators of their children, and with governing body members to foster appreciation of different gifts that build up a learning and faith community and strengthen academic excellence. (*Lay Catholics in Schools*, 78) The Catholic school should pay especially close attention to the quality of interpersonal relations between teachers and students, ensuring that the student is seen as a person whose intellectual growth is harmonized with spiritual, religious, emotional, and social growth. (*The Catholic School on the Threshold of the Third Millennium,* 18)

8. Accessible to All Students
By reason of their evangelizing mission, Catholic schools should be available to all people who desire a Catholic school education for their children. (*Gravissimum Educationis*, 6; *Code of Canon Law,* Canons 793 #2; *Renewing Our Commitment to Catholic Elementary and Secondary Schools in the Third Millennium*, Introduction;) Catholic schools in concert with the Catholic community should do everything in their power to manage available resources and seek innovative options to ensure that Catholic school education is geographically, programmatically, physically, and financially accessible.

9. Established by the Expressed Authority of the Bishop

Canon Law states, “Pastors of souls have the duty of making all possible arrangements so that all the faithful may avail themselves of a Catholic education” (*Code of Canon Law*, Canon 794). Bishops need to put forward the mission of Catholic schools, support and enhance the work of Catholic schools, and see that the education in the schools is based on principles of Catholic doctrine. (John Paul II, *Pastores Gregis,* 52) Catholic schools have a formal and defined relationship with the Bishop guided by a spirituality of ecclesial communion, and should work to establish a relationship marked by mutual trust, close cooperation, continuing dialogue, and respect for the Bishop’s legitimate authority. (*Code of Canon Law,* Canon 803 #1 and #3; Miller, 33)

Structure of the Document

This mathematics standards-based curriculum represents the compilation of on-going research into current mathematics teaching best practice, thoughtful consideration of teaching and assessment methods used in the Archdiocese, and collaboration and consultation with teachers and experts in the field of mathematics in developing content and student learning objectives.

The standards for mathematics instruction in the Archdiocese of Hartford are divided by grade level and then outlined sequentially by quarter. Within each grade level, with the exception of Algebra I, High School Geometry, High School Algebra II, and Precalculus, there are four **domains**:

- Number Theory, Operations, and Algebraic Thinking
- Measurement
- Geometry
- Data Analysis, Statistics and Probability

Domains are large groups of related standards. Standards from different domains may sometimes be closely related.

The **ARCHDIOCESAN STANDARDS** listed at the beginning of each grade level originated as restatements of the National Council of Teachers of Mathematics (NCTM) Learning Standards and have been merged with the Common Core State Standards (CCSS). The Archdiocesan standards reflect the spirit of the Common Core with its instructional shifts while maintaining the integrity of rigor and relevance of our standards. "These movements served to highlight the many variables of teaching and learning and the centrality of curriculum development… these perennial movements reflect an attempt to mirror the tradition of excellence that has been the hallmark of the Catholic schools in the United States while, at the same time, serving as a reminder to us that we must continue to live out our tradition in a new century highlighting those very same variables." (Krebbs, 2012)

Standards are the primary instructional targets that outline essential topics and skills that students must know and be able to do by the end of each respective grade and high school. Following each set of standards are key focus skills and essential questions that describe overarching outcomes. **Student objectives** reflect broad concepts that reflect what students should understand and master. **Enabling outcomes** are bulleted skills that outline what students should specifically be able to do and demonstrate mastery of, in order to achieve the broader student objectives. Teachers are expected to integrate mathematics in all subject areas and to protect instructional time to ensure a greater depth of understanding in the area of mathematics across all grade levels.

The student objectives outlined in each quarter represent an *instructional plan* for the year. This curriculum provides guidance to teachers regarding content to be addressed at each specific grade level and in each quarter. The standards are comprehensive and cover a wide range on the curricular spectrum. *Therefore, it is recommended that teachers and administrators identify essential, core curriculum content that is aligned with the provided ****Benchmarks for Critical Foundations in Mathematics*** *and emphasizes enduring understandings, reinforces essential skills and procedures, and includes student interests.* Content must be taught for depth of understanding rather than coverage of objectives. As schools meet in their **professional learning communities,** conversations should be had regarding the use the standards, the use of testing data including formative data, summative data, and standardized test data in order to effectively and efficiently inform instructional planning to meet the needs of each student, and to discuss best practices.

Daily standards-based lesson planning enables educators to align curriculum and instruction with standards, as they have been adapted by this Archdiocese, thereby keeping the goals of our students in mind. The purpose of standards-based curriculum is to empower all students to meet new, challenging standards of education and to "provide them with lifelong education…that equips them to be lifelong learners." (Fullan, 2006)

The premise of this curriculum is based upon the NCTM Standards, CCSS, and the Purpose and Vision for Catholic School Education in the Archdiocese of Hartford. Instruction should be modeled upon those standards, both in content and in style. Classrooms should incorporate a learning environment that values problem solving in real life situations and encourages the active participation of the students in the learning process. Instruction should engage students in the learning process rather than allowing them to be the passive recipients of information. Providing opportunities for students to think creatively and critically, to communicate and collaborate, and to integrate technology will prepare them for their future.

Each introduction of a new skill or concept should be developed with the idea that *knowing mathematics is doing mathematics*. Associated learning activities should arise from problem situations. Learning should include opportunities for appropriate project work, group and individual assignments alike, discussions between teachers and students, practice, and teacher exposition. In addition, students should have frequent opportunities to formulate problems and questions that arise from their own interests. Small group work can be both collaborative and cooperative, ensuring that each individual student is assessed and not the "group." The ultimate goal of group work should be to enable the student to become a more independent thinker.

Accountable Talk in Mathematics

Instructional programs from prekindergarten through grade 12 should enable all students to--

- organize and consolidate their mathematical thinking though **communication;**
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Just as students are required to read, write, and speak about what they have learned in the language arts and other content areas, so should this be the practice in mathematics. As students are asked to **communicate** about the mathematics they are studying ("Accountable Talk"), they gain insights into their thinking. In order to communicate their thinking to others, students naturally reflect on their learning and organize and consolidate their thinking about mathematics. *The ability to write about mathematics* should be particularly nurtured across the grades.

By working on problems with classmates, students also have opportunities to see the perspectives and methods of others. They can learn to understand and evaluate the thinking of others and to build on those ideas. They may benefit from the insights of students who solve the problem using a visual representation. Students need to learn to weigh the strengths and limitations of different approaches, thus becoming critical thinkers about mathematics. **Differentiating instruction plays a paramount role in this determination and in planning *daily learning objectives*.**

Problem Solving

The mastery of problem solving strategies is a critical component of 21st century literacies that students must advance to become productive members of a global society. As the curriculum evolves during the course of the school year, teachers are urged to note the various problem-solving strategies cultured and integrated throughout the enabling outcomes. Some of these strategies may include: draw text and electronic pictures, make a chart, table, graph, use manipulatives, choose a method/operation, write number sentences, make a model, identify patterns, solve a simpler problem, act it out, use logical reasoning, guess and check, or work backwards.

Resources/Strategies/Cross Curricular Connections

Each grade level of the document ends with two or three tables. On the primary and intermediate levels, there is a resource table for reading-math connections. On all levels, there are two additional tables, one that suggests teaching and learning strategies and another that lists suggestions for cross curricular and Catholic social teachings connections. Strategies and integration activity suggestions are minimal as these sections are designed to be expounded upon by the classroom teacher.

Sequence

The Archdioceses of Hartford Mathematics Curriculum Standards is organized in sequence by quarter. Teachers and administrators should determine what is core or essential for all learners and what is supplemental or enrichment aspects of the curriculum, using the Archdiocesan Benchmarks as a guide. Each mathematics teacher should become familiar with the objectives for the preceding as well as the following grade, and have a good overall picture of the sequence of instruction throughout the twelve grades.

Grades Seven/Eight, Algebra I and Secondary

It is our goal that all students will complete Algebra I by the end of eighth grade. Completion of algebra in grade eight affords students the possibility of completing five years of secondary mathematics before college. Nurturing the expectation that all students will take Algebra I eliminates the possibility of inequality and untapped potential that may result from accelerating only a few students into Algebra. However, if a student needs a stronger foundation in standard grade 7 or grade 8 math to ensure a successful year of Algebra I in high school, that is the recommended course for that student. Benchmark assessments are encouraged to be given at the end of grade 6 to determine readiness for a grade 7 pre-algebra course. The Archdiocesan Algebra Readiness Test should be given at the end of grade 7 to determine readiness for a grade 8 algebra course. The Archdiocesan Algebra I End-of-Course Assessment should be given to students completing the 8th grade Algebra I course. The most important goal is that Catholic school students in the Archdiocese of Hartford have a rich and challenging middle school math experience; one that builds on the foundation of algebraic thinking begun and nurtured through the primary and intermediate levels.

The secondary school structure is very different from its primary, intermediate, and middle school counterparts. This section of the document, more than any other, is based on Archdiocesan Curricular Standards, the Common Core State Standards, as well as the Massachusetts State Standards which also exceeds CCSS. The structure follows a more general framework to accommodate both required and elective math courses and the various ability levels offered. Each school should design their course syllabi using these standards as the foundation for teaching and learning and year-end goals.

Use of Technology

As in all areas of curriculum, technology can and should enhance learning of mathematics. There are countless website resources for student exploration and practice and for assisting teachers in lesson planning. Interactive white boards provide powerful opportunities for motivating and challenging students in the study of mathematics. Calculators, too, are a valuable tool in math instruction. The National Council of Teachers of Mathematics, in its position statement on the use of technology, states:

> **Calculators, computer software tools, and other technologies assist in the collection, recording, organization, and analysis of data. They also enhance computational power and provide convenient, accurate, and dynamic drawing, graphing, and computational tools. With such devices, students can extend the**

range and quality of their mathematical investigations and encounter mathematical ideas in more realistic settings.

In the context of a well-articulated mathematics program, technology increases both the scope of the mathematical content and the range of the problem situations that are within students' reach. Powerful tools for computation, construction, and visual representation offer students access to mathematical content and contexts that would otherwise be too complex for them to explore. Using the tools of technology to work in interesting problem contexts can facilitate students' achievement of a variety of higher-order learning outcomes, such as reflection, reasoning, problem posing, problem solving, and decision making. Technologies are essential tools within a balanced mathematics program. Teachers must be prepared to serve as knowledgeable decision makers in determining when and how their students can use these tools most effectively.
(http://www.nctm.org/about/position_statements/position_statement)

While these tools do not replace the need to compute mentally, do reasonable paper and pencil computation, and learn facts; calculators, computers, hand held data devices, etc. must be accepted as valuable tools for learning and teaching mathematics. Their effectiveness depends on the ability of students to recognize reasonable answers.

Additionally, technological tools enable students to extend their problem solving ability beyond their knowledge of paper and pencil computation. This increases their math power. These tools also free students from tedious computation and allow them to concentrate on problem solving, both the posing and the solving of problems.

Calculators in grades 5 through 8 should include the following features: square root, reciprocal, exponent, +/- keys, algebraic logic, and constants. Some use of graphing calculators in Algebra I is recommended.

All textbook publishers provide interactive websites for teachers, students, and parents. (These are listed in the Approved Programs and Texts list published by the Office of Catholic Schools.) Almost all have the availability of online texts and often have proprietary software in conjunction with their series. This support includes lesson plans for teachers, practice and challenge opportunities for students, as well as activities for parents. In addition, both web and software resources offer a variety of choices in assessment tools. Teachers should investigate, select and use these resources carefully.

Instructional Resources

The materials needed to support math instruction **on every level** should reflect three sequential components of learning. First, the student needs multiple concrete experiences that illustrate a mathematical principle or process. Students should be given access to manipulatives (both physical and virtual) – those materials that can be organized, categorized, combined, separated, changed – that provide varied concrete experiences of mathematical thinking and processes. These materials should include, but are not limited to: unifix cubes, geoboards, spinners, coins, counters, pattern blocks, fraction pieces, algebra tiles, compasses, scales, scissors, rulers, protractors, graph paper, grid/dot paper. Samples of these are found in the teachers resources of any math text.

Once the student has recognized a general pattern, materials and instruction are provided to help the student explain, describe, and represent what has taken place. The manipulation of materials is represented as an algorithm, a written record of thinking. Finally, the student develops the ability to apply concrete experiences to real world and abstract situations, often as problem solving.

Each student should have adequate resources to learn. For most schools, these resources would include a text either in print or electronic form. The text should be chosen from the Archdiocesan Approved Programs and Texts list. Additional classroom resources might include student workbooks, computer generated practice materials and games designed to develop mathematical thinking.

All schools should have a membership in the National Council of Teachers of Mathematics.

*ADH Benchmarks for Critical Foundations in Mathematics

The following **Benchmarks for Critical Foundations in Mathematics** should be used to guide classroom curricula, mathematics instruction, and assessments. They should be interpreted flexibly, to allow for the needs of students and teachers. *For our purposes,* ***proficient*** *is defined as 80-85% mastery.*

The major goals for K-8 mathematics education should be:

- Proficiency with **whole numbers**
- Proficiency with **fractions** (including decimals and percents)
- Proficiency with particular aspects of **geometry and measurement**

Fluency with Whole Numbers

1. By the end of grade 3, students should be proficient with the addition and subtraction of whole numbers.
2. By the end of grade 4, students should be proficient with multiplication and division of whole numbers.

Fluency with Fractions

1. By the end of grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
2. By the end of grade 5, students should be proficient with comparing fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals.
3. By the end of grade 5, students should be proficient with multiplication and division of fractions and decimals.
4. By the end of grade 5, students should be proficient with all operations involving positive and negative integers.
5. By the end of grade 5, students should be proficient with all operations involving positive and negative fractions.
6. By the end of grade 6, students should be able to solve problems involving percent, ratio, and rate, and extend this work to proportionality.

Geometry and Measurement

1. By the end of grade 3, students should be able to solve problems involving perimeter.
2. By the end of grade 4, students should be able to solve problems involving the area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
3. By the end of grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area.
4. By the end of grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.
5. By the end of grade 8, students should be able to analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.

National Council of Teachers of Mathematics

Mathematics Standards

Instructional programs from pre-kindergarten through grade twelve should enable all students to:

1. Students understand numbers, ways of representing numbers, relationships among numbers, and number systems
2. Students understand meanings of operations and how they relate to one another
3. Students compute fluently and make reasonable estimates
4. Students understand patterns, relations, and functions
5. Students represent and analyze mathematical situations and structures using algebraic symbols
6. Students use mathematical models to represent and understand quantitative relationships
7. Students analyze change in various contexts
8. Students analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
9. Students specify locations and describe spatial relationships using coordinate geometry and other representational systems
10. Students apply transformations and use symmetry to analyze mathematical situations
11. Students use visualization, spatial reasoning, and geometric modeling to solve problems
12. Students understand measurable attributes of objects and the units, systems, and processes of measurement
13. Students apply appropriate techniques, tools, and formulas to determine measurements
14. Students formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
15. Students select and use appropriate statistical methods to analyze data

Archdiocesan Standards

16. Students will use their study of math to make data-driven moral decisions and to promote justice in the world.

We must expect all of our students to learn mathematics well beyond what we previously expected. We need all students to be more proficient than in the past, and we need many more students to pursue careers based on mathematics and science.

Seely, Cathy, NCTM http://www.nctm.org/news/pastpresident/2005_03president.htm

Common Core State Standards for Mathematical Practice

Standards for Mathematical Practice	Summary
1. Make sense of problems and persevere in solving them.	• Find meaning in problems. • Analyze, predict, and plan solution pathways. • Verify answers. • Students ask themselves the question: "Does this make sense?"
2. Reason abstractly and quantitatively.	• Make sense of quantities and their relationships in problems. • Create coherent representations of problems.
3. Construct viable arguments and critique the reasoning of others.	• Understand and use information to construct arguments. • Make and explore the truth of conjectures. • Justify conclusions and respond to arguments of others.
4. Model with mathematics.	• Apply mathematics to problems in everyday life. • Identify quantities in a practical situation. • Interpret results in the context of the situation and reflect on whether the results make sense.
5. Use appropriate tools strategically.	• Consider the available tools when solving problems. • Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a Web site, and other technological tools).
6. Attend to precision.	• Communicate precisely to others. • Use clear definitions, state the meaning of symbols, and are careful about specifying units of measure and labeling axes. • Calculate accurately and efficiently.
7. Look for and make use of structure.	• Discern patterns and structures. • Can step back for an overview and shift perspective. • See complicated things as single objects or as being composed of several objects.
8. Look for an express regularity in repeated reasoning.	• When calculations are repeated, look for general methods, patterns, and shortcuts. • Be able to evaluate whether an answer makes sense.

GRADE 1 ADH STANDARDS:

Number Theory, Operations, and Algebraic Thinking (NOA)	
NOA.1.1	Understand numbers, place value, ways of representing numbers, and relationships among numbers
NOA.1.2	Understand and apply place value, properties of operations, and the relationship between addition and subtraction
NOA.1.3	Represent and solve problems involving addition and subtraction.
NOA.1.4	Add and subtract fluently within 20
NOA.1.5	Understand patterns in various contexts
NOA.1.6	Use mathematical models to represent and understand quantitative relationships
NOA.1.7	Analyze change of quantity and quality using patterns
NOA.1.8	Use addition and subtraction with commutative and associative properties to determine equivalence and solve
NOA.1.9	Use fractions to draw conclusions about fairness and equity of resources
Measurement (M)	
M.1.1	Understand standard and nonstandard units of measurement
M.1.2	Apply appropriate techniques and tools to solve problems including measurements, time, and money
M.1.3	Understand measurable attributes of objects and the units, systems, and processes of measurement
Geometry (G)	
G.1.1	Analyze characteristics and properties of two and three dimensional geometric shapes
G.1.2	Apply transformations and use symmetry to analyze mathematical situations
G.1.3	Use visualization, spatial reasoning, and geometric modeling to solve problems
Data Analysis & Probability (DP)	
DP.1.1	Select and use appropriate methods to collect, organize, and analyze data
DP.1.2	Develop and evaluate inferences and predictions that are based on data
DP.1.3	Understand and apply basic concepts of probability

GRADE 1 KEY FOCUS SKILLS:

- **DEVELOP AN UNDERSTANDING OF ADDITION AND SUBTRATION; ADD AND SUBTRACT FLUENTLY WITHIN 20**
- **APPLY ADDITION AND SUBTRACTION SKILLS AND CONCEPTS TO REAL WORLD PROBLEMS**

MATH FACTS ARE CUMULATIVE SKILLS AND MUST BE CONSISTENTLY REINFORCED THROUGH *PRACTICAL, AUTHENTIC APPLICATIONS* THROUGHOUT THE YEAR TO ACHIEVE MASTERY.

GRADE 1: ESSENTIAL UNDERSTANDINGS	GUIDED QUESTIONS -- What Students Need to Know
Numbers are used to name, count, and place objects in order.	How are numbers used to name, count, and place objects in order? How do fractions describe parts of a whole? How does position of a digit in a multi-digit number determine its value?
Estimation is used to approximate exact values.	Why is it helpful to be able to count from a given number instead of from one? How do people know if an estimate is reasonable?
A variety of methods are used to develop understanding and skill in estimation and computation.	When is it appropriate to use mental math, pencil and paper, calculators, or computers to do rounding and computation? How are concrete materials used to model and solve mathematical problems?
Patterns are used to investigate, understand, and describe the world.	What kinds of patterns can be found in natural and human-designed environments? How are number patterns used to solve problems?
Patterns and number relationships are used to understand and solve problems.	In an open sentence, how can the unknown number be determined from the known numbers and the operation?
Number operations are used to solve problems.	How do characteristics of a problem lead to a choice of a number operation? What rules/properties influence the ways operations can be used to solve problems? How is subtraction related to addition?
Measurement is used to communicate about size and shape	How are length, weight, time, and money used to describe and compare things? When is it useful to estimate measurements? What kinds of tools are used to find measurements?
Geometric shapes and positions of objects are used to describe the world.	How can three-dimensional shapes be combined to create a new shape? How do plane figures differ from solid figures? What are examples of geometric shapes and relationships in architecture, art, and nature?
Data can be used to predict outcomes and support conclusions.	How can data be organized? How can data be used to draw conclusions and make decisions? What factors need to be considered in making a prediction?

G1:Q1

TOPIC	OBJECTIVE	ENABLING OUTCOMES: The students will…
Addition & Subtraction to 12	To count by groups, add one more to groups, and compare groups. (NOA.1.4; 1.8)	• **Count, read, write, order, compare, expand** and **represent** numbers to 120 • **Count** on from a given amount, orally and with models • **Count** back from 20 • **Identify** one more and one less than a number • **Plot** numbers to 100 on a number line • **Identify** and **use** zero
	To develop and apply fact families using inverse relationships. (NOA 1.1; 1.4; 1.2)	• **Memorize** addition and related subtraction facts to 12 • **Check** subtraction with addition • **Relate** the inverse relationship of addition and subtraction facts to 12 • **Apply** addition and subtraction facts to real world situations • **Solve** problems involving addition and subtraction
	To add by counting and combining and subtract by separating, comparing, or counting on or back. (NOA.1.1; 1.3, 1.8) To represent the result of counting, combining, and separating sets of objects using number sentences. (NOA.1.4; 1.6) To examine attributes of objects and describe their relationships. (NOA.1.5;1.7)	• **Represent** addition and subtraction on a number line • **Model** real-life situations that involve addition and subtraction of whole numbers using objects, pictures, and open sentences • **Identify, describe, extend,** and **create** patterns • **Describe** how specific patterns are generated

G1:Q2

TOPIC	OBJECTIVE	ENABLING OUTCOMES: The students will…
Place Value	To represent and order 2 digit numbers using the base ten place value system. (NOA.1.4; 1.8)	• **Identify** number words to ten • **Identify** ordinal position of objects first through tenth • **Identify** ordinal words to tenth • **Identify** and **name** place values • **Use** place value models to **identify** tens and ones • **Identify** and **name** place values to hundreds place • **Identify** 10 more and 10 less than a number
	To describe quantitative relationships and develop benchmark representations. (NOA.1.6)	• **Estimate** quantity of items in a group • **Estimate** and **describe** quantity with benchmark amount such as 1, 10 and 100.

	To identify and represent quantities as equivalent or non-equivalent. (NOA.1.6; 1.8)	• **Demonstrate** equivalence using models • **Identify** and **use** symbols of inequality (<, >) • **Identify** and **apply** symbol of equality (=) • **Balance** simple number sentences by finding the missing numbers
Addition & Subtraction to 20	To analyze change of quantity and quality using patterns. (NOA.1.7) To develop and apply fact families using inverse relationships. (NOA.1.1; 1.2; 1.4)	• **Skip count** by 2,5,10 • **Represent** even and odd numbers concretely as pairs and leftover ones • **Identify** even and odd numbers to 100 • **Describe** relationships between quantities with familiar contexts using ratios: one desk has four legs, two desks, eight, etc.
	To build on previous understanding of addition and subtraction to develop quick recall of basic addition and subtraction facts (NOA 2.3) To understand and describe functional relationships in real-world situations. (NOA.1.8) To create and solve one step story and picture problems. (NOA.1.3) To describe quantitative relationships and develop benchmark representations. (NOA.1.6)	• **Memorize** addition and related subtraction facts to 20 • **Identify** missing addends (sums to 20) • **Solve word problems** that call for addition of three whole numbers whose sum is less than or equal to 20 • **determine** the missing addend or subtrahend in a problem (3 + _ = 5 or _ - 2 = 3) • **understand** subtraction as an unknown addend problem • **add and subtract using commutative and associative properties** • **Identify** functional number relationships • **Choose** addition or subtraction to complete function tables • **Choose** the correct operation in a word problem (+,-) • **Identify** reasonable answers to problems that reflect real-world experience. • **Selec**t a reasonable answer to a problem reflecting a change in place value (i.e., 5, 50, 500)

G1:Q3

TOPIC	OBJECTIVE	ENABLING OUTCOMES: The students will…
Money	To determine and compare coin values (M.1.2; 1.3) To express monetary value in oral and written forms (M.1.2; 1.3) To recognize, identify, and trade equivalent sets of coins (M.1.2; 1.3) To express monetary value in oral and written forms (M.1.2; 1.3) To solve problems involving money (M.1.2; 1.3)	• **Name** a penny, nickel, dime, quarter and dollar bill • **Identify** the value of a penny, nickel, dime, quarter and dollar bill • **Use** the cents sign (¢) • **Determine** and **compare** values of sets of coins • **Trade** with sets of pennies and dimes • **Count** and **show** money to one dollar • **Use** dollar sign ($) with decimal point • Solve problems involving real world use of money • **Add and subtract** money to 12 cents
Time	To use calendars and clocks to	• **Tell and/or show** time to the hour using both analog and digital clocks • **Tell and/or show** time to the half hour using both

Measurement	measure and record time (M.1.2; 1.3) To plan and sequence events (M.1.1)	analog and digital clocks • **Write** time in standard notation • **Estimate** elapsed or projected time in terms of an hour or a minute • **Identify** days of the week, months of the year, current year • **Use** a calendar to **identify** dates • **Read and write** the date • **Identify** the number of days in a month • **Use** a calendar to **identify** dates and **sequence** events • **Describe** time in terms like: today, yesterday, next week, last week, tomorrow • **Estimate and compare** the length of time needed to complete tasks using terms like longer or shorter
	To measure through direct comparison and repetition of units (M.1.2; 1.3) To use standard units to communicate measure (M.1.2; 1.3) To use concrete examples to make estimates and to determine and describe the reasonableness of answers to measurement problems (M.1.2; 1.3) To measure through direct comparison and repetition of units (M.1.2; 1.3) To use standard units to communicate measure (M.1.1) To use concrete examples to make estimates and to determine and describe the reasonableness of answers to measurement problems (M.1.2; 1.3) To measure through direct comparison and repetition of units (M.1.2; 1.3)	• **Recognize** and **apply** nonstandard units of measure • **Identify** inch and foot as standard customary units • **Demonstrate** approximate inch, approximate foot • **Compare** lengths of given objects using "longer" and "shorter" • **Estimate** and **measure** length and height in non-standard units • **Identify** centimeter as standard metric measure • **Estimate** and **measure** length and height in inches and centimeters • **Identify** cup, pint, quart and pound as standard customary units • **Identify** liter as standard metric unit • **Compare** capacity using "more" or "less" • **Compare** mass of objects using a balance scale • **Compare** volume/capacity of given containers using concrete materials, i.e., water, sand, beans, etc. • **Solve** problems using forms of measurement

G1:Q4

TOPIC	OBJECTIVE	ENABLING OUTCOMES: The students will…
Geometry	To examine attributes of objects and describe their relationships. (G.1.1) To describe, name and interpret relative direction, location, proximity, and position of objects (G.1.3)	• **Sort, classify, and order** objects by size, number, and other properties • **Identify** points inside, outside, or on a figure • **Use** the descriptive terms: top, bottom, left, right, near, far, up, down, above, below, next to, close by • **Sort** and **describe** plane figures (square, circle, rectangle, triangle)

	To classify plane figures and solids by common characteristics including examples with change of position (G.1.3)	• **Identify** plane figures • **Identify** common objects in the environment that depict plane figures • **Count** corners and sides of plane figures • **Explore and identify** solid figures (cube, cone, cylinder, sphere)
	To recognize and use geometric relationships to solve problems (G.1.3)	• **Identify** figures having the same size and shape • **Identify** open or closed figures • **Explore** lines of symmetry • **Create** shapes and design with symmetry • **Build and draw** two and three dimensional shapes • **Draw** shapes from memory (i.e., draw a triangle) • **Predict** the results of putting together and taking apart two- and three-dimensional shapes
Fractions	To identify and compare equal parts of a whole (NOA.1.9) To partition a set of objects into smaller groups with equal amounts. (NOA.1.9) To identify and compare equal parts of a whole (NOA.1.9)	• **Identify** equal parts of a whole • **Make** a whole of equal sized parts of familiar objects • **Identify** halves and quarters using models • **Identify** half of a small set of objects considered to be the whole. • **Read, write,** and **identify** 1/2, 1/3, 2/3, 1/4, 2/4, 3/4 • **Differentiate** halves, thirds and fourths from other fractional parts • **Recognize** and **model** halves, thirds, and fourths of a whole or set; understand that decomposing a whole or set into more equal shares creates smaller shares • **Identify** *fractions* on a number line • **Compare** parts of a whole object and estimate whether they are closer to zero, one half or one whole
Data & Graphs	To determine the likelihood of certain events through simple games and experiments (DP.1.3) To collect, organize, and describe data (DP.1.1) To analyze data in tables and graphs (DP.1.1; 1.2)	• **Identify** events as certain, possible or impossible (If a bowl is filled with red jelly beans, is it possible to pick a red jelly bean from the bowl? A green one?) • **Observe, record, graph**, and **describe** the results of simple probability activities and games • **Read** and **Use** data from a graph, table, glyphs (coded pictures), and/or picture • **Make and interpret** a real object, picture, and bar graphs • **Make and interpret** a tally chart • **Pose** questions to collect data • **Conduct** simple surveys to gather data • **Choose** and **Use** various methods to organize information including lists, systematic counting, sorting, graphic organizers, and tables • **Use** comparative language to describe/interpret data in tables and graphs • **Use** a Venn diagram and other graphic organizers to sort items
Number Theory	To add by counting and combining and subtract by **separating, comparing, or counting on or back. (NOA.1.8)**	• **Develop, describe, choose** and **use** strategies to add and subtract one- and two-digit numbers

		• **Add** and **subtract** 2 digit numbers without regrouping • **Add** 1 and 2 digit numbers with three addends (column addition) • **Add** and **subtract** 3 digit numbers without regrouping • **Add** and **subtract** using commutative and associative properties

VOCABULARY

Number Theory	equal to; place names: ones , tens hundreds
Whole Numbers	add; addend; addition sentence ; count on; difference; doubles; fact families; minus; number sentence; plus ; related facts; subtraction sentence; sum; turn-around fact; +, -, =
Fractions	fourth ; fraction; half; part; third ; whole
Estimation	between; estimate; greater than; less than
Algebra	even; number; odd; pair; pattern; <, >, =
Geometry	angles; corners ; face; inside/outside; left and right; open and closed figures; plane figures ; sides; solid figures; symmetry; top and bottom
Measurement	length/height: centimeter; foot; inch ; longer/shorter ; metric ; standard ; Capacity: cup ; liter ; pint; quart; more/less Money: cent ¢; dime ; dollar $; nickel ; penny; quarter Temperature; thermometer Time: half hour ; hour ; o'clock
Data Analysis, Statistics, Probability	bar graph; data; graph; greater than/less than/equal to; less/more; possible/impossible; certain; table; tally; Venn diagram; vertical

Resources for Grade One Math Literacy Connections	
Strand	***Book Title***
Number Theory	***Over in the Meadow*, Langstaff and Rojankowsky. San Diego: Harcourt Brace, 1957.** ***Hold Tight Bear*, Rod Maris, New York: Delacorte, 1989.** ***Yellow Ball*, Molly Bang, New York: Morrow, 1991.** ***The Enormous Turnip*, Kathy Parkinson.** ***The Crickets from Mouse Soup*, Arnold Lobel.** ***Maurice Goes to School*, B. Wiseman. *Bandaids*, Shel Silverstein.** ***Animal Numbers*, Bert Kitchen, New York: Dial, 1987.** ***The Bicycle Race*, Donald Crews, New York: Greenwillow, 1985.** ***M&M Counting Book*, Barbara Barbieri McGrath.** ***Bunches and Bunches of Bunnies*, by Louise Matthews.** ***Eating Fractions*, Bruce McMillan. New York: Scholastic, 1991.** ***The Doorbell Rang*, Pat Hutchins.** **New York: Scholastic, 1986.**
Algebra	***Ten in a Bed*, Mary Rees, Boston: Little Brown, 1988.** ***Mouse Count*, Ellen Stoll Walsh, San Diego: Harcourt Brace, 1990.** ***Bat Jamboree*, Kathi Appelt, Morrow, 1996.** ***Frog and Toad are Friends*, Arnold Lobel, *Harper Trophy*, 1970.**
Geometry	***Circles, Triangles, and Squares*, Tana Hoban. New York: Macmillian, 1974.** ***The Most Wonderful Eggs in the World*, Melme Heine.** ***The Greedy Triangle*, Marilyn Burns.** ***Grandfather Tangs Story*, Ann Tompert.**
Measurement	**"A List" from *Frog and Toad Together*, Arnold Lobel.** ***Mud for Sale*, Brenda Nelson.** ***If You Give a Mouse a Cookie*, Laura Joffee Numeroff. New York: Harper Collins 1985.** ***Inch by Inch*, Leo Lionni. New York: Astor-Honor, 1962.** ***Is It Larger, Is It Smaller*, Tana Hoban, New York: Green Willow, 1985.**

Suggested Cross Curricular and Catholic Social Teaching Links

Grade One

- Students measure the growth of classroom plants, record their observations and talk about taking care of God's creation. (Science, Math, Religion, Written language)
- Students keep a graph of sunny/cloudy days and write prayers thanking God for both. (Math, Science, Religion, Language Arts)

GRADE 2 ADH STANDARDS

Number Theory, Operations, and Algebraic Thinking (NOA)

NOA.2.1	Understand and apply place value, ways of representing numbers, properties of operations, and the relationship between addition and subtraction
NOA.2.2	Represent and solve problems involving addition and subtraction.
NOA.2.3	Add and subtract fluently within 20
NOA.2.4	Use *fractions* to draw conclusions about the fairness and equity of resources
NOA.2.5	Understand patterns; represent and analyze mathematical problems using algebraic properties of addition and subtraction
NOA.2.6	Use mathematical models to represent and understand quantitative relationships
NOA.2.7	Analyze change of quantity and quality using patterns
NOA.2.8	Use addition and subtraction with commutative and associative properties to determine equivalence and solve
NOA.2.9	Use fractions to draw conclusions about fairness and equity of resources

Measurement (M)

M.2.1	Use appropriate tools to measure and estimate length, volume, and capacity in standard and nonstandard units.
M.2.2	Relate addition and subtraction to length, time, and/or money
M.2.3	Solve problems including measurement, time, and/or money

Geometry (G)

G.2.1	Analyze characteristics and properties of two and three dimensional geometric shapes and develop mathematical arguments about relationships
G.2.2	Apply transformations and use symmetry to analyze mathematical situations
G.2.3	Use visualization, spatial reasoning, and geometric modeling to solve problems

Data Analysis and Probability (DP)

DP.2.1	Select and use appropriate methods to collect, organize, and analyze data
DP.2.2	Develop and evaluate inferences and predictions that are based on data
DP.2.3	Understand and apply basic concepts of probability

GRADE 2 KEY FOCUS SKILLS:

- **ADD AND SUBTRACT FLUENTLY WITHIN 20; APPLY SKILLS TO REAL WORLD APPLICATIONS**
- **ADD AND SUBTRACT WITHIN 100 USING PAPER AND PENCIL**
- **EXTEND UNDERSTANDING OF BASE-TEN NOTATION**

MATH FACTS ARE CUMULATIVE SKILLS AND MUST BE CONSISTENTLY REINFORCED THROUGH *PRACTICAL, AUTHENTIC APPLICATIONS* THROUGHOUT THE YEAR TO ACHIEVE MASTERY.

GRADE 2 - Essential Understandings	Guided Questions -- What Students Need to Know
Place value is used to determine the value of each digit in the number.	How does position of a digit in a multi-digit number determine its value? When adding two- or three-digit numbers, what happens when the two digits in the ones column equal a number greater than 10?
Number operations are used to solve problems.	How do characteristics of a word problem lead to a choice of a number operation? What rules/properties influence the ways operations can be used to solve problems?
A variety of methods are used to develop understanding and skill in rounding and computation.	When is it appropriate to use mental math, pencil and paper, and calculators or computers to do estimation and computation? How are concrete materials used to model and solve mathematical problems?
Whole figures can be divided into fractional parts.	Why is it possible for equal shares of the same whole to have different shapes?
Patterns and number relationships are used to understand and solve problems.	How are number patterns used to solve problems? In an open sentence, how can the unknown number be determined from the known numbers and the operation?
Measurement is used to communicate about size, shape, time, and money.	When is it useful to estimate measurements? What kinds of tools are used to find measurements? What strategies can be used to measure and compare objects?
Geometric shapes are used to describe the world.	What are examples of geometric shapes and relationships in architecture, art, and nature? How can shapes and relationships be used to create things?
Data can be used to predict outcomes and support conclusions.	What kind of data can be collected? How can data be organized? How is data used to draw conclusions and make decisions?

G2:Q1

TOPIC	OBJECTIVE	ENABLING OUTCOMES: The students will…
Addition and Subtraction to 20	To represent the result of counting, combining and separating sets of objects using number sentences (NOA2.1, 2.2) To develop fact families using inverse relationships (NOA 2.8, 2.3, 2.4) To build on previous understanding of addition and subtraction to develop quick recall of basic addition and subtraction facts (NOA 2.3)	• **Model** real-life situations that involve addition and subtraction of whole numbers, using objects, pictures and open sentences • **Write** related fact families for addition and subtraction • **Relate** the inverse relationship of addition and subtraction facts to 20 • **Memorize** addition and related subtraction facts to 20 • **Solve problems** and **apply** addition and subtraction facts to real world situations
	To analyze how both repeating and growing patterns are generated (NOA 2.6) To identify, describe, create, and extend a number of patterns (NOA 2.5, 2.7)	• **Describe** attributes and relationships of objects • **Sort, classify**, and **order** objects and numbers based on one and two attributes and describe the rule used • **Translate** the same pattern from one representation (such as color) to another representation (such as shape) • **Describe** counting and number patterns • **Explore** and **solve** problems involving simple number patterns. • **Identify** objects with common or different attributes • **Identify** missing objects in a pattern
Place Value Add and Subtract 2-Digit Numbers	To identify and represent quantities as equivalent or nonequivalent (NOA 2.8) To use number sentences to represent quantitative relationships (NOA 2.7)	• **Read** and **write** number words to one hundred or beyond; read and write numerals to 999 • **Identify** and **use** symbols of inequality (<, >,) • **Use** concrete, pictorial, and verbal examples to **demonstrate** an understanding that = is a relationship that indicates equivalence • **Demonstrate** balance or equivalence using models • **Identify** and **use** symbols of inequality (‹, ›) • **Identify** and **use** symbol of inequality (≠)
	Students will analyze change in quantity and quality using patterns. (NOA 2.5) To use concepts based on patterns and place values to add and subtract (NOA 2.5) To identify functional number relationships in real-world situations (NOA 2.1)	• **Balance** simple number sentences by finding the missing numbers • **Identify** missing numbers to 20 in addition and subtraction sentences and **justify** the answer • **Determine** and **justify** the missing addition/subtraction signs in addition and subtraction sentences • **Identify** and **justify** missing numbers in addition and subtraction sentences
	To represent the result of counting, combining and separating sets of objects using number sentences (NOA 2.1) Students will identify and use equivalent representations of numbers to estimate and compute. (NOA 2.8)	• **Determine** whether a number is even or odd using manipulatives • **Skip count** by 3, 4, and 100 • **Identify** numbers as odd or even • **Identify** number words to one hundred • **Identify** and **name** place values: hundreds, tens and ones • **Identify** ordinal positions and words to twentieth

G2:Q2

TOPIC	OBJECTIVE	ENABLING OUTCOMES: The students will…
Add and Subtract 2-Digit Numbers **Length, Capacity, Volume/Time** **Measurement** **Money**	To use prior understanding of addition and subtraction to develop strategies for multi-digit addition and subtraction (NOA 2.1, 2.2) To develop, discuss, and use efficient, accurate, and various methods to add and subtract multi-digit whole numbers (NOA 2.1, 2.2) To develops fluency with efficient procedures for adding and subtracting whole numbers, understand why the procedures work, and use them to solve problems (NOA 2.1, 2.2, 2.3) To determine and use various tools and units to estimate and measure (M 2.1) To use measurement to determine and explain relative size of a given object (M 2.1) To identify and generalize relationships between measurable attributes of plane and solid figures (M 2.1) To use standard units and identify examples of measurements in daily life (M 2.1) To recognize, identify and trade sets of equivalent coins (M 2.3) To express monetary values in oral and written forms (M 2.3)	• **Add** and **subtract** 2 digit numbers with regrouping • **Add** 1 and 2 digit numbers with 3 addends – column addition • **Choose** addition or subtraction to complete functions tables • **Identify** missing addends with 2 digit numbers • **Choose & justify** the correct operation in a word problem (+, -) • **Check** subtraction with addition • **Round** numbers to the nearest 10 • **Round** to estimate sums of two digit numbers • **Use** estimation strategies that result in reasonable answers to a problem • **Build** fluency with addition and subtraction by applying standard algorithms to real world applications • **Tell and/or show** time to the half hour using both analog and digital clocks • **Tell, write, and show** time to the quarter hour, to five and one minute intervals • **Estimate and/or compute** elapsed or projected time in terms of an hour or a minute • **Use** *A.M.* and *P.M.* appropriately • **Recognize and apply** non standard units of measure • **Estimate** and **measure** length and height in centimeters and inches • **Compare** and **order** objects according to length • **Identify** cup, pint, quart, liter and gallon and **relate** to their use in real life • **Compare and order** objects according to capacity and/or weight • **Demonstrate** balance or equivalence using models • **Identify** pound as a unit of measure and **relate** use in real life • **Supplemental: Read** Fahrenheit and Celsius thermometers • **Count and show** money to one dollar • **Find** equivalent sets of coins • **Use** dollar sign • **Use** decimal point in writing money amounts • **Make** change up to $1.00

G2:Q3

TOPIC	OBJECTIVE	ENABLING OUTCOMES: The students will…
Number Theory and Operations **Plane and Solid Figures** **Spatial Relationships** **Graphs Data Analysis** **Probability**	To recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another (NOA 2.6) To develop, discuss, and use efficient, accurate, and various methods to add and subtract multi-digit whole numbers (NOA 2.1, 2.2) To classify and identify plane figures and solids by common characteristics (G 2.1) To identify shapes as the same where there are changes in position (G 2.2) To collect, organize, and describe data (DP 2.1) To pose questions to be answered through collection and analysis of data (DP 1.2) To determine the likelihood of certain events through games and simple experiments (DP 1.3**)**	• **Use** estimation strategies that result in reasonable answers to a problem • **Build** fluency with addition and subtraction by applying standard algorithms to real world applications • **Relate** solid figures to common items • **Recognize, name, compare, and sort**: cube, cylinder, cone sphere, rectangular prism, and pyramid • **Identify, model/construct** geometric solids by the attributes: face, edge, and vertices • **Describe** the relationship between plane and solid figures • **Describe** plane and solid figures by number of sides and/or faces • **Classify** plane figures by size and shape • **Identify** corners, sides, and points inside and outside of a figure • **Identify** and **create** open and closed figures • **Identify** congruent figures • **Find** the area of squares and rectangles by **modeling** and **counting** square units • **Demonstrate** ways to fill a region with different shapes • **Model** and **identify** the perimeter of a polygon • **Recognize, apply** and **manipulate** slides, flips and turns • **Explore, identify** and **draw** lines of symmetry in simple shapes and forms • **Recognize** and **create** simple figures and drawings with symmetry • **Identify** translations, rotations, and reflections • **Read and interpret** vertical graphs, pictographs • **Conduct** simple surveys to gather data • **Create** a tally chart using given data • **Create** simple (picture, bar) graphs from given data • **Use** a Venn diagram and other graphic organizers to sort items • **Demonstrate** and **explain** survey findings • **Use** range and mode to explain data • **Identify** events as certain, possible or impossible, fair or unfair (If a bowl is filled with red M&M's, is it possible to pick a red M&M from the bowl? A green M&M?) • Predict sample data

G2:Q4

TOPIC	OBJECTIVE	ENABLING OUTCOMES: The students will…
Fractions	To create portions of equal size to illustrate *fractions* (NOA 2.3) To apply fractions to draw conclusions about fairness of resources (NOA 2.10)	• **Read, write** and **identify** halves, thirds and fourths • **Identify** more than one equal part of a region, area, or object • **Describe** the significance of a numerator and denominator • **Compare** parts of whole object and describe them as closer to zero, one half, or one whole • **Identify** *fractions* on a number line (halves, thirds and fourths) • **Read, write** and **identify** all *fractions* • **Compare** unit fractions • **Compare** fractions with like denominators • **Use** visual models to identify and compare *fractions* • **Identify** and **model** fractional parts of a set • **Model** equivalent fractions (using manipulatives, pictures, graphics, etc.) • **Place** *fractions* (halves, thirds, and fourths) on a number line
Number Theory **Place Value**	To represent three digit numbers as groups of hundreds, tens, and ones in the base ten number system (NOA 2.1) To use concepts based on patterns and place values to add and subtract (NOA 2.3, 2.5, 2.7)	• **Demonstrate** place values using models • **Write** expanded numerals in standard form • **Expand numerals** by identifying the value of each digit in its place • **Count, order, compare**, and **expand** numerals to 999 • **Identify** and **name** place values to the thousands place • **Add** and **subtract** 3 digit numbers without regrouping • **Add** and **subtract** 3 digit numbers with regrouping • **Round** numbers to the nearest hundred • **Subtract** 3 digit numbers with regrouping through zeroes • **fluently add and subtract** within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction
Multiplication and Division	To describe the relationship between multiplication and division (NOA 2.1)	• **Relate** skip counting and repeated addition to multiplication. • **Draw** arrays to model multiplication • **Explore** products to 25 • **Use** models to **demonstrate** division (Make equal groups and use repeated subtraction.) • **Illustrate** repeated addition and subtraction on a number line • **Use** arrays to **relate** multiplication and division
Enrichment	To recognize and explore Roman numerals	• **Identify** Roman numerals I, V, and X • **Read** and **write** Roman numerals to 30

<u>VOCABULARY</u>	**Number Theory**	Attributes; between; compare; digit; just before; just after; number line; ordinal; pattern; roman numeral; strategy; thousands s
	Whole Numbers	Column; factor; product; regrouping
	Fractions	Fourths; halves; thirds
	Estimation	Estimation; number line
	Algebra	Associative; balance; commutative; equivalent/non-equivalent; quantity
	Geometry	Angle; area; face; flip; perimeter; plane figure; polygon; similar; symmetry; solid; turn side
	Measurement	Analog; area; Celsius/ Fahrenheit; change; Degree; Digital; dollar; elapsed time; gallon; gram; half dollar; half past; kilogram; measure; meter; ounce; perimeter; pound; price; time; total; yard
	Data Analysis, Statistics, Probability	equal to; fair/unfair; greater/less than; horizontal; mode; predict; range; Venn diagram; vertical

Resources for Grade Two Math Literacy Connections

Strand	***Book Title***
Number Theory	***A Birthday Basket for Tia*, by Pat Moran** ***Ocean Parade*, by Patricia McCarthy** ***Numbers of Things*, by Helen Oxenbury** ***A Thousand Pails of Water*, by Ronald Roy** ***Two Hundred Rabbits*, by Lonzo Anderson and Adrienne Adams** ***Even Steven & Odd Todd Making Sense of Census 2000*, Scholastic** ***Each Orange had Eight Slices*, by Paul Giganti** ***Ninety-nine Pockets*, by Jean Myrick** ***How many Snails*, by Paul Giganti** ***How Many Feet in the Bed*, by Diane Hamry** ***One Hundred Hungry Ants*, by Elinor Pinczes.** ***Fractions are Parts of Things*, by Richard Dinnis** ***How Many Ways Can you Cut a Pie*, by Jane Belk Moncure**
Geometry	***The Village of Round and Square Houses*, by Ann Grifalconi** ***The Button Box*, by Margarette S. Reid**
Measurement	***How Big is a Foot*, by Rolf Myller** ***On a Hot, Hot Day*, by Nicki Weiss** ***Farmer Mack Measures his Pig*, by Toni** ***Bargain for Frances*, by Russell Hoban** ***Penelope Gets Wheels*, by Esther Peterson**

	Where the Sidewalk Ends,* by Shel Silverstein** ***Clocks and More Clocks, by Pat Hutchins ***Alexander Who Used to be Rich Last Sunday*,** **by Judith Viorst**

Suggested Cross Curricular and Catholic Social Teaching Links **Grade Two**
• Students draw maps of their community/communities (neighborhood, parish, school yard, etc.), write address numbers in different ways (One Hundred Grant St., 100 Grant St.). (Art, Social Studies, Math) • Students graph ways in which people in communities help one another and ways in which they can help their communities (family, school, parish, and neighborhood)). (Religion, Social Studies, Math) • Students make string phones with a paper cup at each end; they record and graph sounds heard at 10 ft, 20 feet, etc. (Science, Math) • Students plan a food drive. (Religion, Math, Health) • Students compare pieces of string, one cut 53 inches, the length of a dinosaur's foot, the other the length of the student's foot, and write a paragraph describing their conclusions. (Science, Math) • Students work together to plan a bus route from their homes to school and compare lengths of routes with one another. (Social Studies, Math)

GRADE 3 ADH STANDARDS

Number Theory, Operations, and Algebraic Thinking (NOA)	
NOA.3.1	Use place value understanding and properties of operations to perform multi-digit arithmetic.
NOA.3.2	Understand properties of multiplication and the relationship between multiplication and division.
NOA.3.3	Solve problems involving all four operations, and identify and explain patterns in arithmetic.
NOA.3.4	Compute fluently through 12s tables and apply to real world situations
NOA.3.5	Develop an understanding of fractions as numbers
NOA.3.6	Extend whole numbers, place value, patterns, and notations to include decimals; relate money to decimals
NOA.3.7	Represent and analyze mathematical situations and structures using algebraic symbols
NOA.3.8	Use mathematical models to represent and understand quantitative relationships
NOA.3.9	Use fractions to draw conclusions about fairness and equity of resources
Measurement (M)	
M.3.1	Apply appropriate techniques, tools and formulas to determine measurements, including time and money
M.3.2	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects
Geometry (G)	
G.3.1	Analyze characteristics and properties of two and three dimensional geometric shapes and develop mathematical arguments about relationships
G.3.2	Understand concepts of area and perimeter and relate to multiplication and addition
G.3.3	Apply transformations and use symmetry to analyze mathematical situations and solve problems
Data Analysis, Statistics, and Probability (DSP)	
DP.3.1	Collect, organize, and display data; select and use appropriate statistical methods to analyze data
DP.3.2	Develop and evaluate inferences and predictions that are based on data
DP.3.3	Understand and apply basic concepts of probability

GRADE 3 KEY FOCUS SKILLS:

- **DEVEVLOP UNDERSTANDING OF MULTIPLICATION AND DIVISION; MULTIPLY WITHIN 100**
- **ADD/SUBTRACT WITHIN 1000**
- **APPLY MATH COMPUTATIONS AND CONCEPTS TO REAL WORLD SITUATIONS**
- **DEVELOP AN UNDERSTANDING OF FRACTIONS; READ, WRITE, AND IDENTITY FRACTIONS**

MATH FACTS ARE CUMULATIVE SKILLS AND MUST BE CONSISTENTLY REINFORCED *AND APPLIED TO REAL WORLD SITUATIONS* THROUGHOUT THE YEAR TO ACHIEVE MASTERY.

GRADE 3 - Essential Understandings	Guided Questions -- What Students Need to Know
Mathematics can be used to describe, understand, and communicate about the world in order to solve problems and make decisions. Characteristics of a situation or problem influence the choice of numbers, operations, strategies, and tools. Patterns aid description, understanding, and communication about the world. Patterns and number relationships can be used to investigate, understand, and solve problems. Measurement allows description, understanding, and communication about the world. Attributes and relationships of shapes, objects, and patterns can be used to describe, understand, and communicate about the world. Data collection and analysis can be used to predict outcomes, solve problems, and make decisions.	What does mathematics reveal about the world? What situations require the use of mathematical understanding? How can patterns and properties of operations be used when adding and subtracting? What is the relationship between multiplication and division? How can strategies be used to determine the reasonableness of an answer? How do the characteristics of a problem influence the choice of numbers, operations, strategies, and tools? How and why are patterns used? How are patterns and number relationships represented with symbols? How are tables and equations used to represent, analyze, and extend patterns? How do patterns help to solve problems and communicate information? What kinds of strategies help to reveal patterns and number relationships? How are tables, graphs, and equations used to discover, analyze, and extend patterns and number relationships? How does the precision required for a measurement influence the choice of strategies and tools? How is understanding and communication about measurement used to solve problems and make decisions? How can objects in the natural and human-designed world be identified and described in geometric terms? How do models and drawings enhance understanding? How can shared attributes help to define categories of shapes? How are models and drawings used in problem solving and design? What factors influence the way data is collected and organized? How is the reliability of data affected by the source, quantity, and method of collection? How is the analysis of data used to solve problems? How is the presentation used to support different kinds of data? Why would one style of graph, chart, or table be more appropriate than another when depicting data?

G3:Q1

TOPIC	OBJECTIVES	ENABLING OUTCOMES: The students will…
Number Theory **Place Value** **Addition & Subtraction of Whole Numbers** **Estimation** **Money**	To represent and order number concepts in verbal and written form (NOA 3.1) To represent four digit numbers as groups of thousands, hundreds, tens, and ones in the base ten number system (NOA 3.1) To use prior understanding of addition and subtraction to develop strategies for multi-digit addition and subtraction (NOA 2.1, 2.2) To develop, discuss, and use efficient, accurate, and various methods to add and subtract multi-digit whole numbers (NOA 2.1, 2.2) To develops fluency with efficient procedures for adding and subtracting whole numbers, understand why the procedures work, and use them to solve problems (NOA 2.1, 2.2, 2.3) To represent the result of counting, combining and separating sets of objects using number sentences (NOA 3.1, 3.3) To identify and represent quantities that are equivalent or non-equivalent (NOA 3.8) To identify and use equivalent representations of numbers based on place value patterns to estimate and compute (NOA 3.6) To express monetary values in oral and written forms (M 3.1) To recognize, identify and trade sets of equivalent coins (M 3.1)	• **Read** and **write** number words to one thousand • **Identify** and **name** place values to the thousands place • **Expand numerals** by identifying the value of each digit in its place • **Write** expanded numerals in standard form • **Read** and **write** numerals to 9999 • **Count, order, compare**, and **expand** numerals to 9999 • **Identify** and **name** place values to the hundred thousands place • **Read** and **write** numerals to 999,999 • **Count, order, compare**, and **expand** numerals to 999,999 • **Add** and **subtract** fluently through 12s tables; **apply** to real world applications • **Solve problems** and **apply** addition and subtraction facts to real world situations • **Add** and **subtract** basic facts within 1000 • **Add and subtract** six digit numbers • **Add** and **subtract** 3 digit numbers with regrouping • **Add** three or more addends (column addition) • **Create** story problems using number sentences • **Balance** number sentences by finding the missing numbers • **Apply** patterns and properties of operations as strategies to add and subtract including **commutative, associative, and distributive properties** • **Identify** missing addends with 2 digit numbers • **Identify** and **use** symbols for greater than (›),less than (‹) and not equal (≠) • **Describe** the relationships of place values to regrouping • **Subtract** 3 digit numbers with regrouping through zeroes • **Choose** and **justify** the correct operation in a word problem (+, -) • **Identify** numbers as odd or even • **Round** numbers to the nearest hundred; nearest thousand • **Use** front-end estimation • **Estimate** sums and differences and describe the method of estimation • **Select** reasonable answers to an estimation problem • **Describe** and **use** estimation strategies that can identify a reasonable answer to a problem when an estimate is appropriate • **Subtract** amounts of money less than a dollar

	To solve problems involving money (M 3.2)	from amounts greater than a dollar • **Use** decimal point in writing money amounts • **Find** equivalent sets of coins • **Identify** half dollars • **Make** change to a dollar • **Add** and **subtract** sums of money less than a dollar in columns **aligning** decimal points

G3:Q2

TOPIC	OBJECTIVES	ENABLING OUTCOMES; The students will…
Multiplication and Division Concepts	To use concepts based on patterns and place value to multiply and divide (NOA 3.2) To analyze change in quantity and quality using patterns (NOA 3.1) To use properties of whole numbers to maintain equivalence (NOA 3.8)	• **Relate** skip counting and repeated addition to multiplication • **Draw** arrays to model multiplication • **Skip count** by 3, 4, and 100 • **Explore** and **describe** multiplication fact patterns • **Identify, express** and **apply** the zero properties of multiplication • **Identify, express** and **apply** the commutative, associative and identity properties of addition and multiplication
Multiplication and Division Facts	To identify functional number relationships in real-world situations (NOA 3.8) To identify and represent quantities that are equivalent or non-equivalent (NOA 3.8)	• **Illustrate** repeated addition and subtraction on a number line • **Choose** multiplication or division to complete functions tables • **Memorize** multiplication facts and related division facts through **12 times table** • **Apply** multiplication facts to **solve** real world problems • **Apply** properties of operations as strategies to multiply and divide including commutative, associative, and distributive properties • **Identify and justify** missing numbers in multiplication and division facts • **Use** mental math to **multiply** by 10, 100, and 1000
Fractions	To develop understanding of fractions as numbers (NOA 3.9)	• **Understand** a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts**; understand** a fraction a/b as the quantity formed by a parts of size $1/b$. • **Understand** a fraction as a number on the number line; represent fractions on a number line diagram • **Represent** a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line. • **Represent** a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

G3:Q3

TOPIC	OBJECTIVES	ENABLING OUTCOMES: The students will…
Multiplication by 1-Digit Numbers	To represent the result of counting, combining and separating sets of objects using number sentences (NOA 3.3)	• **Multiply** two and three digit numbers by a one digit number • **Recognize** and **apply** the distributive property of multiplication
Division by 1-Digit Numbers	To demonstrate equivalence using properties of whole numbers (NOA 3.7) To use estimation strategies that result in reasonable answers to a problem (NOA 3.7) To identify and use equivalent representations of numbers based on place value patterns to estimate and compute (NOA 3.6)	• **Recognize** when estimation is an appropriate problem-solving strategy • **Model** and **interpret** division with remainders • **Multiply** and **divide** money using single digit multipliers/divisors. • **Estimate** products and quotients and the method of estimation • **Use** compatible numbers to **make** reasonable estimates • **Use** clustering to estimate sums • **Divide** with 2-digit dividends and 2-digit quotients • **Record** division using an algorithm (long division) • **Use** benchmarks to understand the relative magnitude of numbers • **Determine** and discuss the reasonableness of an answer and explain why a particular estimation strategy will result in an over or underestimate • **fluently divide** within 100 • **use** multiplication and division within 100 to **solve** word problems in situations involving equal groups, arrays, and measurement quantities
Fractions	To represent *fractions* by sharing portions of equal size (NOA 3.5) To explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. (NOA 3.5) To use models and number lines to compare *fractions* (NOA 3.5) To model and identify mixed numbers (NOA 3.5) To construct and use models to add and subtract like *fractions* (NOA 3.5)	• **Model** equivalent *fractions* (using manipulatives, pictures, graphics, etc.) • **Read, write** and **identify** all *fractions* • **Identify** and **model** fractional parts of a set • **Find** fractional parts of numbered groups • **Use** visual models to identify and compare *fractions* • **Compare** *fractions* with like denominators • **Compare** unit *fractions* • **Compare** proper *fractions* with unlike denominators • **Compare** two fractions with the same numerator or the same denominator by reasoning about their size. • **Recognize** that comparisons are valid only when the two fractions refer to the same whole. • **Record** the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. • **Identify** mixed numbers • **Add** and **subtract** like *fractions* using models • **Apply** fractions to draw conclusions about fairness and equity of resources
Decimals	To extend whole number place value patterns, models, and notations to include *decimals* (NOA 3.6)	• **Model** and **write** *decimals* in tenths and

	To express equivalent relationships between *decimals* and *fractions* whose denominator is a multiple of ten (NOA 3.6, 3.7)	hundredths • **Relate** money (pennies and dimes) to *decimals* • **Compare** and **order** *decimals* of tenths and hundredths • **Locate** *decimals* on a number line • **Count** by tenths and hundredths • **Write** *fractions* with denominators of 10 or 100 as *decimals*

G3:Q4

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Time **Graphs** **Data Analysis** **Measurement**	To determine the likelihood of certain events through games and simple experiments (DSP 3.2) To determine and use various tools and units to estimate and measure (M 3.1) To use standard units and identify and express examples of measurement in daily life (M 3.2) To represent and order time concepts in verbal and written form (M.1) To collect, organize and describe data (DSP 3.1) To identify functional number relationships in real world situations (NOA 3.3) To pose questions to be answered through collection and analysis of a data set (DSP 3.2) To describe features of a data set (DSP 3.1) To determine and use various tools and units to estimate and measure (M 3.2) To use measurement to determine and explain relative size of a given objects and measures (M 3.2) To use standard units and identify and express examples of measurement in daily life (M 3.2)	• **Identify** events as more likely, equally likely, less likely • **Express** probability in verbal and numerical terms • **Use** results of experiments to predict future events • **Calculate** probability of an event • **Estimate and/or compute** elapsed or projected time in terms of an hour or a minute using a clock • **Use** *A.M.* and *P.M*. appropriately • **Tell, write, and show** time to the quarter hour, to five and one minute intervals • **Use** a schedule, calendar, and/or a timeline to measure elapsed time • **Tell** time in two ways (minutes before the hour and minutes after the hour) • **Identify** conversion factors for seconds, minutes, hours, and days • **Identify** ordinal words to thirty-first (calendar-related) • **Create** simple (picture, bar) graphs from given data • **Create** a tally chart using given data • **Read** and **interpret** tally charts, frequency tables, bar graphs, and pictographs • **Use** a variety of graphic organizers to sort items • **Create** diagrams and charts to solve problems • **Draw** Venn diagrams to illustrate given data • **Read** and **interpret** line graphs • **Conduct** surveys to gather data • **Demonstrate** and **explain** survey findings • **Use** range and mode to explain data • **Calculate** mean and use to explain data • **Identify** and **use** median to explain data • **Estimate** and **measure** length and height in inches, feet, and yards • **Estimate** and **measure** length and height in centimeters and meters • **Choose** an appropriate unit to estimate length or distance (foot, yard, mile) • **Measure** to the nearest half and quarter inch • **Estimate** and **measure** length and height in

Geometry **Problem Solving**	To use measurement to determine and explain relative size of a given objects and measures (M 3.1) To classify or identify plane figures and solids by common characteristics (G 3.3) To identify shapes as the same where there are changes in position (G 3.1, 3.2, 3.3) To recognize and use geometric relationships to solve problems (G 3.1, 3.2, 3.3) **To identify characteristics of a situation or problem influence the choice of numbers, operations, strategies, and tools**. (NOA 3.7)	millimeters, decimeters, kilometers • **Memorize** conversions for inches, feet, yards • **Identify** the conversions for feet, yards and miles • **Identify** cup, pint, quart, gallon and **apply** to real life • **Identify** pound and ounce as units of measure and **relate** use in real life • **Identify** a liter as 1000 milliliters • **Identify** liter and **apply** to real life • **Compare and order** objects according to capacity & weight • **Identify** conversion factors in the metric system • **Read** Fahrenheit and Celsius thermometers and **describe** temperatures as hot, warm, or cold • **Recognize, name, compare, and sort**: cube, cylinder, cone sphere, rectangular prism, and pyramid • **Describe** plane and solid figures by number of edges and/or faces • **Describe** the relationship between plane and solid figures • **Identify and draw** points, lines, line segments, and rays • **Classify** angles as right, acute or obtuse • **Identify, compare** and **contrast** intersecting, perpendicular and parallel lines • **Identify, describe, classify** and **draw** polygons: quadrilaterals, pentagons, hexagons, octagons and **classify** triangles according to sides and angles • **Identify** congruent figures • **Compute** the perimeter of a polygon • **Find** the area of squares and rectangles by **modeling** and **counting** square units • **Identify** ways to tile or tessellate a region or shape using various polygons • **synthesize number and operation concepts to solve complex, multi-step word problems using all four operations** • **assess the reasonableness of answers using mental computation and estimation strategies including rounding**

Resources for Grade Three Math Literacy Connections	
Strand	***Book List***
Number Theory	***How Much is a Million*, David M. Schwartz. New York: Morrow, 1985** ***Anno's Mysterious Multiplying*, Jar, Philomel Books, 1983** ***Too Man Kangaroo Things to Do*, Harper Collins, 1996** ***2X2= Boo a Set of Spooky Multiplication Stories*, Holiday House, 1995** ***Charlotte's Web*, E.B. White** ***The 329th Friend*, Marjorie Weinman Sharman, New York: Macmillian Publishers, 1992** ***Sideways Stories from Wayside School*, Louis Sacher. New York: Camelot, 1985** ***Let's Investigate Estimating*, Marion Smoothey, Marshall Cavendish Corporation, 1995** **Gator Pie, Louise Matthews. Dodd Mead 7 Co.** ***Wayside School is Falling Down*, Louis Sacher. NY: Lothrop, Lee & Shephard, 1989** ***Fractions are Parts of Things*, J. Richard Dennis. NY: Harper Collins Children's Books, 1972**
Algebra	***Caps for Sale*, Esphyr Slobodkina Scholastic** **The *I Hate Mathematics*! Book by Marilyn Burns. Little, Brown and Co., 1975** ***20,000 Baseball Cards Under the Sea*. John Buller & Susan Schade. NY: Random House, 1991.** ***Goldilocks and the Three Squares***
Geometry	***A Light in the Attic* (Shapes, p. T1), Shel Silverstein, Harper & Row** ***The Greedy Triangle*, Marilyn Burns: Scholastic, 1994** ***Right Angles: Paper Folding Geometry*, Jo Phillips: Thomas Crownwell Co., 1992.** ***Grandfather Tang's Story*, Ann Tompert**
Measurement	***$1.00 Word Riddle Book*, Marilyn Burns. Cuisenaire** ***Inch by Inch*, Leo Lionn: Astorhmor, 1960** ***A Quarter from the Tooth Fairy*, Carne Holtzman, Scholastic** ***How Much is that Guinea Pig in the Window?* By Joanne Rocklin, Scholastic Inc.**

Suggested Cross Curricular and Catholic Social Teaching Links

Grade Three

- Students write a paragraph comparing and contrasting two solid figures using words like *face* and *edge*. *(Language Arts, Math)*
- Students read a book like *Selina and the Bear Paw Quilt* and create artwork using patterns. (Language Arts, Art, Math)
- Students create fair and unfair spinners for games and discuss the importance of honesty and justice. (Math, Art, Religion

GRADE 4 ADH STANDARDS

Number Theory, Operations, Algebraic Thinking (NOA)	
NOA.4.1	Use place value understanding and properties of operations to perform multi-digit arithmetic.
NOA.4.2	Understand meanings of the 4 operations and how they relate to one another to solve problems
NOA.4.3	Use numbers and their properties to compute fluently and to estimate measures and quantities reasonably
NOA.4.4	Understand, describe, and apply patterns and functional relationships to real world situations
NOA.4.5	Gain familiarity with factors and multiples
NOA.4.6	Use algebraic symbols to determine equivalence and solve problems
NOA.4.7	Extend understanding of fraction equivalence and ordering.
NOA.4.8	Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
NOA.4.9	Understand decimal notation for fractions, and compare decimal fractions.
Measurement (M)	
M.4.1	Develop and apply appropriate techniques, tools and formulas to estimate and determine measurements
M.4.2	Solve problems involving money, measurement, and conversion of measurements from a larger unit to a smaller unit.
Geometry (G)	
G.4.1	Analyze characteristics and properties of two and three dimensional geometric shapes and develop mathematical arguments about relationships and communicate rationale
G.4.2	Understand concepts of angle and measure angles.
G.4.3	Specify locations and describe spatial relationships using coordinate geometry and other representational systems
G.4.4	Apply transformations and use symmetry to analyze mathematical situations
Data Analysis, Statistics, & Probability (DSP)	
DSP.4.1	Select and use appropriate statistical methods to analyze data
DSP.4.2	Analyze data sets to form hypotheses and make predictions
DSP.4.3	Understand and apply basic concepts of probability
DSP.4.4	Develop and evaluate inferences and predictions that are based on data

GRADE 4 KEY FLUENCIES

- **DEVELOP UNDERSTANDING AND FLUENCY OF MULTI-DIGIT MULTIPLICATION & DIVIDING TO FIND QUOTIENTS INVOLVING MULTI-DIGIT DIVIDENDS**
- **ADD/SUBTRACT WITHIN 1,000,000**
- **DEVEVLOP AN UNDERSTANDING OF FRACTION EQUIVALENCE**
- **BUILD UNDERSTANDING OF ADDITION AND SUBTRACTION OF FRACTIONS WITH LIKE DENOMINATORS; MULIPLY FRACTIONS BY WHOLE NUMBERS**

GRADE 4 - Essential Understandings	Guided Questions -- What Students Need to Know
Mathematics can be used to describe, understand, and communicate about the world in order to solve problems and make decisions. Characteristics of a situation or problem influence the choice of numbers, operations, strategies, and tools. Patterns aid description, understanding, and communication about the world. Patterns and number relationships can be used to investigate, understand, and solve problems. Measurement allows description, understanding, and communication about the world. Geometry has many real-world applications including design, architecture, and art. Data collection and analysis can be used to predict outcomes, solve problems, and make decisions.	What does mathematics reveal about the world? How is mathematics used in the everyday world? What situations require the use of mathematical understanding? How can concrete materials model mathematical situations? Using place value, what does the position of each digit reveal about its value? How do the characteristics of a problem influence the choice of numbers, operations, strategies, and tools? What strategies help determine if a solution is reasonable, accurate, and complete? How and why are patterns used? How are patterns and number relationships represented symbolically? How are tables and equations used to represent, analyze, and extend patterns? How do patterns help to solve problems and communicate information? What kinds of strategies help to reveal patterns and number relationships? What is the meaning of a variable in an equation or number expression? How are strategies used to assess the reasonableness of an answer? How do the characteristics of objects and events influence the choice of measurement strategies and tools? How is the understanding and communication about measurement used to solve problems and make decisions? How do the characteristics of geometric figures influence their use in designs? How are models and drawings used in problem solving and design? How is the analysis of data used to solve problems? How is the presentation of data used or misused to support an outcome or decision?

G4:Q1

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Place Value **Estimation** **Addition, Subtraction, Multiplication, Division Facts**	To represent numbers as groups of millions, thousands, hundreds, tens, and ones in the base ten number system (NOA 4.1) To represent and order number concepts in verbal and written form (NOA 4.3) To use place value concepts, number patterns, and number properties to develop estimation and computation strategies (NOA 4.1) To add and subtract whole numbers written in vertical and horizontal form, choosing appropriately between paper and pencil methods and calculators (NOA 4.2) To recognize and demonstrate equivalence using number properties (NOA 4.6) To use number patterns, basic facts, arrays, and place value models to multiply and divide whole numbers (NOA 4.3)	• **Use** place value models, diagrams, number patterns and number lines to identify, order, round, and compare whole numbers to 100,000,000 • **Identify** and **name** place values to the hundred millions place • **Use** ten as a repeated factor to define place value through hundred millions • **Use** mental math to **multiply** by 10, 100, and 1000 • **Build** place value models, **draw** diagrams and **show** equivalent representations for numbers to 999,999,999 in expanded and regrouped form • **Read, write, count, skip count, order, compare**, and **expand** numerals to 999,999,999 • **Write** expanded numerals in standard form • **Identify** and **name** place values to the hundred billions place • **Read** and **write** number words to one billion • **Round** numbers to the nearest thousand, ten thousand, hundred thousand • **Fluently add and subtract** multi-digit whole numbers using the standard algorithm. • **Multiply** a whole number of up to four digits by a one-digit whole number of operations. • **Find** all factor pairs for a whole number in the range 1-100 • **Recognize** that a whole number is a multiple of each of its factors • **Determine** whether a given whole number in the range 1-100 is a multiple of a given one-digit number • **Determine** whether a given whole number in the range 1-100 is prime or composite • **Apply** problem solving skills in multi-step word problems, using the four operations • **Illustrate and explain** the calculation by using equations, rectangular arrays, and/or area models. • **Fluently calculate and apply** multiplication facts and related division facts through 12×12. • **Apply** basic math facts to real world applications/problems • **Add** and **subtract** 6 digit numbers with and without regrouping • **Use** a calculator to add and subtract large numbers • **Use** front-end estimation • **Choose and justify** the correct operation in a word problem (+,-) • **Identify, express** and **apply** the zero property of multiplication • **Describe** the property of zero in multiplication and its implication in division • **Identify, express** and **apply** the commutative, and associative properties of whole numbers in addition

		and multiplication **to estimate, compute, and solve problems** • **Demonstrate** equivalence with the commutative and associative properties of whole numbers • **Demonstrate** equivalence with the distributive property of whole numbers • **Determine** the proper operation to solve a problem and justify the reasoning • **Demonstrate** the equivalence of both sides of an equation as the same value is added, subtracted, multiplied, or divided on each side • **Differentiate** between algebraic expressions and equations • **Find** missing numbers in number sentences • **Find** missing symbols in number sentences (›), (‹), (=) and (≠) • **Find** missing operation symbols in number sentences • **Relate** multiplication and division to models with groups and rectangular arrays • **Multiply** and **divide** money using single digit multipliers/divisors. • **Solve** equations beginning with the operations inside the parentheses

G4:Q2

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Multiplication **Facts by 1 & 2 Digit Numbers**	To use factors to explore, represent and classify numbers (NOA 4.1) To write equations to express relationships between numbers (NOA 4.6) To recognize, create and extend numerical and geometric patterns, using concrete materials, number lines, symbols, tables and words (NOA 4.4) To use factors to explore, represent and classify numbers (NOA 4.5) To use number patterns, basic facts, arrays, and place value models to multiply and divide whole numbers (NOA 4.1) To identify whole number properties and apply them to whole number operations and	• **Multiply** two two-digit numbers, using strategies based on place value and the properties • **Memorize** and **apply** divisibility rules for 2,5, 10 • **Square** a whole number • **Represent** in pictorial form a 2x2 square • **Identify** the written form n^2 • **Multiply** two and three digit numbers by a one digit number with regrouping • **Use** exponents to the power of 2 • **Use** equations to describe the rules for number patterns • **Use** equations to model word problems • **Use** calculators to explore and create number patterns • **Explore** and **describe** multiplication fact patterns • **Describe** and write the rule for number, color, rhythmic and symbolic patterns • **Identify** and **use the inverse** relationships of multiplication and division to solve and check problems • **Solve** practical problems and extend patterns involving 10 and 100 more and less than a number • **Recognize** and **identify** prime and composite numbers to 100

Division	algorithms (NOA 4.2) To use place value concepts, number patterns, and number properties to develop estimation and computation strategies (NOA 4.3) To use number patterns, basic facts, arrays, and place value models to multiply and divide whole numbers (NOA 4.1) To use factors to explore, represent and classify numbers (NOA 4.5)	• **Extend** and **compare** arithmetic and geometric sequences • **Make** generalizations about patterns and relationships and **test** those generalizations • **Multiply** to find special products with multipliers that are multiples of 10, 100, 1000 • **Multiply** four-digit numbers by a one-digit multiplier, two and three digit numbers by a two-digit multiplier • **Divide** three-digit dividends by multiples of 10 • **Divide** three-digit dividends by a one-digit divisor to find quotients of two or three places with zeros and remainders • **Record** division using an algorithm (long division) • **Divide** multiples of 10, 100,1000 and 10,000 by multiples of 10 • **Identify** and **use the inverse** relationships of multiplication and division to solve and check problems • **Model** and **interpret** division with remainders • **Calculate** quotients with and without remainders for 2-, 3-, and 4-digit dividends and 1-digit divisors, based on place value, the properties of operations, and/or the relationship between multiplication and division • **Use** arrays and **explore** using the distributive property [10 x (4+5) = (10 x 5) + (10 x 4)] to estimate, multiply and divide two and three digit numbers by one-digit factors • **Recognize and apply** the distributive property of multiplication • **Use** compatible numbers to **make** reasonable estimates • **Estimate** products and quotients and **describe** the method of estimation • **Describe** and **use** estimation strategies that can identify a reasonable answer to a problem when an estimate is appropriate • **Use** clustering to estimate sums • **Determine** and discuss the reasonableness of an answer and explain why a particular estimation strategy will result in an over or underestimate • **Write** and solve multi-step word problems involving estimation • **Divide** four-digit dividends by a one digit divisor to find three and four digit quotients with zeros and remainders • **Divide** two- and three-digit dividends by two-digit divisors to find one digit quotients with and without remainders • **Use** order of operations to evaluate arithmetic expressions with parentheses

		• **Draw** factor trees • **Identify** the Least Common Multiple (LCM) given pairs of numbers less than or equal to 10 • **Identify** the Greatest Common Factor (GCF) given pairs of numbers up to 81

G4:Q3

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Fractions and Probability **Fractions**	To model, identify, compare *fractions*, and express them in equivalent forms (NOA 4.7) To extend whole number place value patterns, models, and notations to include *decimals* (NOA 4.9) To extend place value concepts and number properties to addition and subtraction of decimal numbers (NOA 4.9) To compute with *fractions* (NOA 4.8) To extend understanding of fraction equivalence and ordering (NOA 4.7)	• **Read, write** and **identify** all *fractions* • **Identify** and **model** fractional parts of a set • **Find** fractional parts of numbered groups • **Use** division to find a fractional part of a set • **Identify** and **find** the simplest form of a fraction • **Write** *fractions* in lowest terms • **Identify and model** equivalent *fractions* (using manipulatives, pictures, graphics, etc.) • **Find and generate** *fractions* that are equivalent using models • **Find** equivalent *fractions* using multiplication and division • **Identify** mixed numbers • **Use** visual/virtual models to identify and compare *fractions* • **Use** models to **change** an improper fraction to a mixed number • **Locate** and **place** *fractions* on a number line • **Apply** the concepts of Greatest Common Factor and Least Common Multiple to *fractions* • **Use** the Least Common Multiple to identify the lowest common denominator of a set of *fractions* • **Add** and **subtract** like *fractions* • **Solve** problems involving addition and subtraction of *fractions* with like denominators • **Compare** proper *fractions* with unlike denominators • **Add** and **subtract** *fractions* with unlike denominators • **Add and subtract** two *fractions* where one denominator is a multiple of the other • **Explain why** a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the numbers and sizes of the parts differ even though the two fractions themselves are the same size; use this principle to recognize and generate equivalent fractions. • **Compare** two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. • **Recognize** that comparisons are valid only when the two fractions refer to the same whole. • **Record** the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual

Decimals **Ratios**	To extend place value concepts and number properties to addition and subtraction of decimal numbers (NOA 4.9)	fraction model. • **Recognize** that comparisons are valid only when the two fractions refer to the same whole • **Recognize and convert** improper fractions and mixed **numbers** • **Decompose** a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation • **Model, read** and **write** *decimals* in tenths and hundredths • **Locate** *decimals* on a number line • **Count** by tenths and hundredths • **Annex** zeroes to create equivalent decimal numbers • **Write** decimal numbers to express *fractions* with denominators of 10 and 100 • **Relate** *decimals* in tenths to *fractions*, and mixed numbers • **Compare** and **order** *decimals* of tenths and hundredths (use symbols <, >, =, and ≠) • **Relate** money (pennies and dimes) to *decimals* • **Round** decimal numbers to the nearest tenth and whole number • **Round** decimal numbers to the nearest hundredth • **Estimate** decimal sums and differences using rounding • **Construct** and **use** models and pictures to add and subtract *decimals* • **Add** and **subtract** *decimals* to hundredths • **Model, read** and **write** *decimals* to thousandths place in standard form and as number words • **Identify** place value in decimal numbers and write *decimals* in expanded form. (EX. 61.34 = 60 + 1 + 0.3 + 0.04) • **Use** models and pictures to estimate reasonable answers when adding or subtracting *decimals*, *fractions*, and mixed numbers • **Write** and solve multi-step word problems with *fractions*, including problems with extraneous information • **Model** and **demonstrate ratios** through the use of concrete objects and pictures using ratios • **Describe** the relationship between *decimals*, *fractions* and *percents* • **Use** models, pictures, and number patterns to **solve** simple problems involving ratio and proportions

G4:Q4

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Fractions	To apply and extend previous understandings of multiplication to multiply a fraction by a whole number (NOA 4.8)	• **Understand** a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent* $5/4$ *as the product* $5 \times (1/4)$*, recording the conclusion by the equation* $5/4 = 5 \times (1/4)$*.* • **Understand** a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express* $3 \times (2/5)$ *as* $6 \times (1/5)$*, recognizing this product as* $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.) • **Solve** word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat* $3/8$ *of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*
	Students understand various mathematical procedures and use them appropriately and accurately (NOA 4.9)	• **illustrate and explain** a calculation by using equations, rectangular arrays, and/or area models • apply problem solving skills in multi-step word problems including problems in which remainders must be interpreted, using the four operations
Money	To express monetary values in oral and written forms (M 4.1) To recognize, identify and trade sets of equivalent coins (M 4.1) To solve problems involving money (M 4.2) To determine and compare coin values (M 4.2)	• **Add** amounts of money less than a dollar to sums greater than a dollar • **Subtract** amounts of money; **Apply** to real world situations • **Apply** and explain a variety of estimation strategies in problem-solving situations to add and subtract money amounts less than $10.00 and two- and three-digit numbers with and without regrouping • **Add** and **subtract** sums of money in columns **aligning** decimal points; • **Round** amounts of money to the nearest dollar
Measurement	To determine and use various tools and units to estimate and measure (M 4.1) To use standard units and identify and express examples of measurement in daily life (M 4.2) To use measurement to determine and explain relative size of a given objects and measures (M 4.2) To determine and use various tools and units to estimate and measure (M 4.1)	• **Choose** an appropriate unit to estimate length or distance • **Estimate, draw,** and **measure** length and height to the nearest inch, half inch, quarter inch and centimeter • **Solve** practical problems that involve estimation and measurement of length, perimeter, and area • **Solve** practical problems that involve estimation and measurement of volume and capacity • **Identify** the conversions for feet, yards and miles • **Estimate** and **measure** length and height in millimeters, decimeters, kilometers • **Define, identify, and use** cup, pint, quart, gallon, liter, milliliter and **apply** to real life • **Define, identify, use** and **relate** benchmarks to ounce

Geometry **Graph and Interpret Data**	To describe geometric properties of plane and solid figures (G 4.1) To identify, draw and describe elements needed to explain spatial relationships (G 4.3) To describe geometric properties of plane and solid figures (G 4.1) To identify and generalize relationships between measurable attributes of plane and solid figures (G 4.1) To determine and use various tools and units to estimate and measure (M 4.1) To use measurement to determine and explain relative size of a given objects and measures (M 4.2) To collect, organize and describe data (DSP 4.1) To represent numerical relationships on a coordinate grid (NOA 4.7)	and gram and **relate** use in real life • **Solve** practical problems that involve estimation and measurement of weight • **Compare and order** objects according to weight • **Identify** conversion factors in the metric system • **Identify** and **use** kilogram and ton • **Build, draw, create, describe,** and **classify** two- and three-dimensional figures • **Sort** polygons and solids by using characteristics such as the relationship of sides (parallel, perpendicular), kinds of angles (right, acute, obtuse), symmetry, and congruence • **Describe** similarities and differences of two and three dimensional shapes in the environment using physical features such as number of sides, number of angles, lengths of sides and straight and curved parts • **Describe** solid figures using faces, edges, and vertices • **Identify and draw** points, lines, line segments, and rays • **Identify, compare** and **contrast** intersecting, perpendicular and parallel lines • **Classify** angles as right, acute or obtuse • **Identify** translations, rotations, and reflections • **Explain** the results of dividing, combining, and transforming shapes and the effects of slides, flips, and turns • **Identify** ways to tile or tessellate a region or shape using various polygons • **Analyze** two-dimensional shapes and determine lines of symmetry and congruence • **Identify, describe** and **classify** triangles according to sides and angles • **Identify, describe, classify** and **draw** polygons: quadrilaterals, pentagons, hexagons, octagons • **Compute** perimeter of a polygon using the formula • **Find** the area of squares and rectangles • **Develop** and **apply** the formula for finding area of squares and rectangles • **Describe** relationships between the lengths of sides of rectangles and their areas and perimeters; **generalize** the patterns as simple formulas • **Find** the volume of rectangular prisms by **modeling** and **counting** cubic units • **Find** strategies for estimating and measuring the perimeters and areas of irregular shapes • **Identify** and **find** the radius and diameter of a circle • **Identify** and **estimate** the circumference of a circle • **Use** a variety of ways to **collect, organize**, **record, analyze**, and **interpret** data and identify patterns and trends

	To use coordinate systems to identify and illustrate spatial location and geometric relationships (G 4.3, 4.4) To recognize, use and simplify arithmetic and algebraic expressions (NOA 4.6) To determine the likelihood of certain events through games and simple experiments (DSP 4.3) To describe features of a data set (DSP 4.2) To pose questions to be answered through collection and analysis of a data set (DSP 4.2)	• **Compute** the mean of a set of data • **Use** range, mean, median, and mode to explain data; **Identify** outliers • **Conduct** surveys to gather data • **Demonstrate** and **explain** survey findings • **Use** technology to **create** spreadsheets and **convert** information into graphs • **Locate** points on a coordinate grid (Quadrant I) using ordered pairs • **Use a table** to explore functions and graph them on a coordinate grid (Quadrant I) • **Make** predictions and defend conclusions based on data • **Express** probability in verbal and numerical terms • **Conduct** probability experiments and express the probability based on possible outcomes • **Express** probability as a fraction • **Identify** possible outcomes of events using combinations where order does not matter

GRADE 4 - Resources for the Grade Four Math Literacy Connections	
Strand	***Book List***
Number Theory	Math Blaster (software) *Remainder of One*, Elinor J. Pinczes, Scholastic, 1993 *Math Curse*, Jim Scieszka & Lane Smith. Viking, 1995, (The Penquin Group) *Anne's Hat Trick*, Philomel Books, 1984 *The Science Book of Numbers*, Jack Challoner, Gulliver Books, 1992 *A Million Fish, More or Less*, Patricia McKissack. Alfred A. Knopf, New York, 1992 *If You Made a Million*, David Schwartz, 1989 *More for me*, Software: Fraction Factory *Gator Pie*, Louise Mathews. Dodd Mead, 1979
Algebra	Game: Battleship, Milton Bradley
Geometry	Math Blaster 2 – Creature Creator Tangrams
Measurements	*How Big Is a Foot?*
Probability, Statistics & Graphing	Microsoft Works / Excel graph survey results

Suggested Cross Curricular and Catholic Social Teaching Links Grade Four
- Students take their heart beats and create equations based on how often their heart beats in a minute, five minutes, etc. (Science, Math) - Students organize a fund raising event for charity setting a goal; they measure their progress toward that goal on a graph in terms of *percents*. (Religion, Math)

GRADE 5 ADH STANDARDS

Number Theory, Operations, Algebraic Thinking (NOA)	
NOA.5.1	Use place value understanding and properties of operations to perform multi-digit arithmetic.
NOA.5.2	Understand meanings of operations and how they relate to one another to solve problems
NOA.5.3	Use numbers and their properties to compute flexibly, fluently, and make reasonable estimates
NOA.5.4	Analyze patterns, relations, and functions
NOA.5.5	Represent and analyze mathematical situations and structures using algebraic symbols to determine equivalence and solve problems
NOA.5.6	Analyze change in various contexts
NOA.5.7	Use equivalent fractions as a strategy to add and subtract fractions.
NOA.5.8	Apply and extend previous understandings of multiplication and division to multiply and divide fractions.
Measurement (M)	
M.5.1	Develop and apply appropriate techniques, tools and formulas to estimate and determine measurements
M.5.2	Convert like measurement units within a given measurement system.
Data Analysis, Statistics, & Probability (DSP)	
DSP.5.1	Formulate questions that can be addressed with data; collect, organize, and display relevant data to answer them using appropriate statistical & graphical methods
DSP.5.2	Analyze data sets to form hypotheses and make predictions
DSP.5.3	Understand and apply basic concepts of probability
DSP.5.4	Develop and evaluate inferences and predictions that are based on data
Geometry (G)	
G.5.1	Analyze properties and characteristics of two-and three-dimensional shapes to describe relationships, communicate ideas and solve problems
G.5.2	Use visualization, spatial reasoning, and geometric modeling to solve problems

GRADE 5 KEY FLUENCIES:

- **MULTI-DIGIT MULTIPLICATION & DIVISION**
- **FLUENCY WITH ADDITION & SUBTRACTION OF FRACTIONS**
- **EXTEND UNDERSTANDING OF MULTIPLICATION & DIVISION OF FRACTIONS**
- **EXTEND DIVISION TO 2 DIGIT DIVISORS; INTEGRATE DECIMAL AND FRACTIONS INTO PLACE VALUE SYSTEM**
- **DEVEVLOP FLUENCY WITH WHOLE NUMBER AND DECIMAL OPERATIONS**

GRADE 5 - Essential Understandings	Guided Questions -- What Students Need to Know
Mathematics can be used to describe, understand, and communicate about the world in order to solve problems and make decisions.	What does mathematics reveal about the world? What situations require the use of mathematical understandings? How does mathematics enable people to work with things they cannot see? How do concrete materials model mathematical situations? What does the position in a multi-digit number reveal about its value?
Characteristics of a situation or problem influence the choice of numbers, operations, strategies, and tools.	How do the characteristics of a situation influence the choice of numbers, operations, strategies, and tools? How is a solution determined to be reasonable, accurate, and complete? Why are comparisons of two fractions only valid when they refer to the same whole?
Patterns aid description, understanding, and communication about the world.	How and why are patterns used? How are patterns and number relationships represented symbolically? What kinds of patterns can be found in natural and human-designed environments? How are tables and equations used to represent, analyze, and extend patterns?
Patterns and number relationships can be used to investigate, understand, and solve problems.	How do patterns help people to solve problems and communicate information? What kinds of strategies help to reveal patterns and number relationships? How are function tables and equations used to discover, analyze, and extend patterns and number relationships?
Measurement allows description, understanding, and communication about the world.	How is measurement used to quantify information about objects and events? How do the characteristics of objects and events influence the choice of measurement strategies and tools? How does the precision required for a measurement influence the choice of strategies and tools? How is the understanding and communication about measurement used to solve problems and make decisions?
Geometry has many real-world applications including design, architecture, and art.	How do the characteristics of geometric shapes and figures influence their use in aesthetic and functional designs? How are geometric shapes and relationships manipulated to create a visual or emotional effect? How are models and drawings used in problem solving and design?
Data collection and analysis can be used to predict outcomes, solve problems, and make decisions.	What factors influence the way data is collected and organized? How is the reliability of data affected by the source, quantity, and method of collection? How is the analysis of data used to solve problems? How is the presentation of data used or misused to support different points of view?

G5:Q1

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Whole Numbers & Place Value **Place Value with Decimals** **Whole Numbers with Decimals** **Integers**	To understand number concepts and use numbers appropriately and accurately (NOA 5.1) To represent numbers in expanded and regrouped forms in the base ten place value system (NOA 5.1) To extend whole number place value concepts to include decimal numbers that are also represented as *fractions* whose denominators are multiples of ten (NOA 5.1) To use place value concepts, number patterns, and number properties to develop and apply estimation and computation strategies (NOA 5.3) To explore numbers less than zero and extend the number line to illustrate integers (NOA 5.1) To use place value concepts and the commutative and associative properties to add and subtract flexibly and fluently (NOA 5.3)	• **Fluently multiply** multi-digit whole numbers using the standard algorithm • **Identify** and **name** place values to the hundred billions place • **Build** place value models, **draw** diagrams and **show** equivalent representations for whole numbers in expanded and regrouped form • **Use** place value models, diagrams, number patterns and number lines to identify, order, round, and compare whole numbers to one billion • **Read, write, count, skip count, order, compare**, and **expand** numerals to one billion • **Write** expanded numerals in standard form; **write** standard form numbers in expanded and word form • **Round** whole numbers to all place values • **Build** models and **describe** tenths and hundredths using equivalent ratio, fraction and decimal notation • **Read** and **write** *decimals* to thousandths place in standard form as number words • **Identify** place value in decimal numbers and write *decimals* in expanded form (EX. 61.34 = 60 + 1 + 0.3 + 0.04) • **Use** models to **extend** whole number place value concepts and patterns to *decimals* • **Compare** and **order** *decimals* to thousandths place from greatest to least and from least to greatest (use symbols ›, ‹, = and ≠) • **Read** and **write** *decimals* to ten thousandths place in standard form as number words • **Use** (greater than or equal, less than or equal) symbols (≥, ≤) • **Round** decimal numbers to the nearest hundredths, tenths, and whole number • **Express** *fractions* with denominators of 10 and 100 as *decimals* • **Annex** zeroes to create equivalent *decimals* • **Relate** *decimals* in tenths and hundredths to *fractions*, mixed numbers, and number words • **Round** *fractions* to nearest half or whole to estimate answers to problems. • **Estimate** decimal sums, differences, products, and quotients using rounding • **Use** benchmarks to understand the relative magnitude of numbers • **Select** and **apply** the most suitable estimation strategy: rounding, clustering, front end (with adjustment, compatible numbers, and compensation) • **Determine** and discuss the reasonableness of an

Distributive Property **Multiplying 1 and 2 Digits** **1 Digit Divisors**	To use number patterns, basic facts, arrays, place value models and the distributive property to multiply and divide (NOA 5.2)	answer and explain why a particular estimation strategy will result in an over or underestimate • **Estimate** decimal quotients using compatible numbers • **Use** a number line to compare and order integers • **Solve** problems involving finding 10,000, and 1000 more or less than a number • **Add** and **subtract** whole numbers (up to 9 digits) presented in both horizontal and vertical form, including column addition. • **Add** and **subtract** *decimals* to the ten thousandths place • **Develop** strategies**,** using place value relationships, inverse operations, and the commutative, associative, and distributive properties to simplify computation with two-, three-, and four-digit numbers and money amounts • **Identify** and **use** the inverse relationships of multiplication and division to solve and check problems • **Determine** the proper operation to solve a problem and **justify** the reasoning • **Express** remainders in division as *fractions* • **Multiply** and **divide** *decimals* by whole numbers • **Use** the short division algorithm (to follow mastery of long division) • **Multiply** and **divide** *decimals* by *decimals* • **Change** a fraction to a decimal using division • **Add, subtract, and multiply, and divide** decimals through the hundredths place using concrete models or drawings and strategies based on place value, properties of operations, rounding, and/or the relationship between addition and subtraction and explain the reasoning • **Use** arrays and explore using the distributive property [10 x (4+5) = (10 x 5) + (10 x 4)] to estimate, multiply and divide two and three digit number • **Recognize and apply** the distributive property of multiplication • **Estimate** products and missing factors using multiples of 10, 100, 1000 • **Use mental math** to multiply by 10, 100, and 1000 • **Use** mental math to **multiply** by multiples of 10, 100, and 1000 • **Multiply** to **find** special products with multipliers that are multiples of 10, 100, 1000 • **Multiply** four digit numbers by a one digit multiplier, two and three digit numbers by a two digit multiplier and three digit numbers by a two digit multiplier • **Describe** the property of zero in multiplication and its implication in division • **Divide** three-digit dividends by multiples of 10 and 100

		• **Divide** multiples of 10, 100,1000 and 10,000 by multiples of 10, 100, and 1000 • **Divide** multi-digit dividends by one and two digit divisors to find multi-digit quotients with zeros and remainders • **Solve** problems involving finding 10, 100. And 1000 more and less than a number • **Determine** the proper operation to solve a problem and **justify** the reasoning

G5:Q2

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Divisibility Rules **Prime and Composite Numbers** **Prime Factorization** **GCF & LCM** **Squaring Numbers** **Fractions** **Operations with Fractions and Mixed Numbers** **Fractions and Reciprocals**	To use factors to explore, represent and classify numbers (NOA 5.2, 5.5) To model, identify, and express equivalent forms of numbers expressed as whole numbers, *fractions* and mixed numbers (NOA 5.7) To add and subtract *fractions* and mixed numbers using models, pictures and number sentences (NOA 5.7) To use models and pictorial representations to develop concepts and methods by which to multiply and divide *fractions* and mixed numbers (NOA 5.8)	• **Memorize** and **apply** divisibility rules for 2,3,5,6,9 and 10 • **Recognize** and **identify** prime and composite numbers to 100 • **Use** rectangular arrays to identify factor pairs and to classify numbers as prime, composite, and perfect squares • **Draw** and **use** factor trees to determine all the factors of a number • **Draw** and **use** factor trees to f**ind** all prime factors and **write** prime factorization of numbers • **Represen**t numbers by using exponents • **Change** exponent form to standard numeral, write as repeated factors and vice versa • **Use** order of operations including exponents • **Identify** the Greatest Common Factor (GCF) given pairs of numbers up to 81 • **Identify** the Least Common Multiple (LCM) given pairs of numbers less than or equal to 10 • **Draw** and **use** factor trees to determine all the factors of a number • **Identify** the written form n^2 • **Represent** in pictorial form a 2x2 square • **Square** a whole number • **Use** exponents to the power of 2 • **Memorize** the perfect squares of numbers from 1 to 15 • **Express** a perfect square in exponent form • **Identify** and **find** equivalent *fractions* • **Locate** and **place** *fractions* and mixed numbers on a number line • **Identify** and **find** the simplest form of a fraction • **Write** *fractions* in lowest terms • **Use** models to **change** an improper fraction to a mixed number • **Find** fractional parts of numbered groups • **Write** *fractions* with a denominator of 100 as percent • **Write** *percents* as *decimals* and *decimals* as *percents* • **Write** *percents* as *fractions* in simplest form • **Construct** and **use** models to a**dd** and **subtract** like

		and unlike *fractions* and mixed numbers • **Use** equivalence and substitution with common denominators when adding and subtracting • **Add** and subtract like and unlike *fractions* and mixed numbers **expressing** answers in simplest form • **Use** models and pictures to estimate reasonable answers when adding or subtracting *decimals*, *fractions*, and mixed numbers • **Use** models to **change** an improper fraction to a mixed number • **Recognize** that multiplication by a unit fraction is equivalent to dividing by the fraction's denominator • **Construct** and **use** models and pictorial representations to multiply common *fractions* and mixed numbers • **Use** models to **divide** whole numbers by *fractions* and *fractions* by whole numbers • **Model** and **describe** when products or quotients with *fractions* and *decimals* can yield a larger or smaller result than either factor • **Multiply** and **divide** *fractions*, whole numbers and mixed numbers • **Subtract** mixed numbers with renaming • Use benchmark fractions and number sense of fractions to **estimate** mentally and **assess** the reasonableness of answers • **Interpret** a fraction as division of the numerator by the denominator • **Interpret** multiplication of fractions as scaling (resizing) by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication • **Explain w**hy multiplying a given number by a fraction greater than 1 results in a product greater than the given number • **Explain** why multiplying a given number by a fraction less than 1 results in a product smaller than the given number • **Recognize** that multiplication by a unit fraction is equivalent to dividing by the fraction's denominator • **Identify** reciprocal numbers • **Apply** reciprocal numbers to division of a whole number by a fraction • **Write** whole number division problems in fraction form and round the fraction form to estimate an answer to a division problem • **Multiply** and **divide** *fractions*, whole numbers and mixed numbers • **Use** cancellation in multiplication of *fractions* • **Solve** real-world problems involving multiplication of fractions and mixed numbers, (e.g., by using visual fraction models or equations)
Measurement	To determine and use various tools and units to estimate and measure (M 5.1)	

	To use measurement to determine and explain the relative size of given objects and measures (M 5.1) To use standard units to identify and express examples of measurement in daily life (M5.1)	• **Interpret** division of a whole number by a unit fraction (e.g., 4 ÷ 1/5 = 20 because 20 x 1/5 = 4) and a unit fraction by a whole number or non-zero number, **compute,** and **apply** to real-world problem solving • **Estimate** and **measure** length and height in millimeters, decimeters, kilometers • **Define, identify, use** and **relate** benchmarks in metric and standard systems • **Use** the appropriate customary and metric units and tools for measuring volume and capacity • **Define, identify, use** and **relate** benchmarks of capacity • **Explain** the difference between mass and weight • **Add** and **subtract** measurements with regrouping recording answers in simplified form • **Identify** and **use** kilogram and ton • **Use** the appropriate customary and metric units and tools for measuring weight • **Define, identify, use** and **relate** benchmarks of weight/mass • **Use** the appropriate customary and metric units and tools for measuring temperature • **Identify** the conversions for feet, yards and miles • **Identify** conversion factors in the metric system • **Compare** and **convert** measures of capacity • **Identify** conversion for pounds and ounces • **Read** Fahrenheit and Celsius thermometers including temperatures below zero • **Find** the change in temperature when one temperature is below zero and the other above

G5:Q3

TOPICS	OBJECTIVES	ENABLING OUTCOMES; The students will…
Simplifying and Evaluating Expressions **Integers and Absolute Value** **Writing and Solving Equations** **Integers and Functions** **Fractions** **Graphing and Equations** **Measurements of Central Tendency** **Ratios-Percents** **Probability**	To recognize, use and simplify arithmetic and algebraic expressions (NOA 5.5) To explore numbers less than zero and extend the number line to illustrate integers (NOA 5.1) To recognize and demonstrate equivalence using number properties (NOA 5.4) To write expressions, equations and inequalities to express relationships between numbers (NOA 5.1) To represent numerical relationships on a coordinate grid (NOA 5.4) To investigate how a change in one variable change in second variable (NOA 5.6) To identify and describe situations with constant or varying rates of change and compare them (NOA 5.6) Use equivalent fractions as a strategy to add and subtract fractions (NOA Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction (NOA To use tables, graphs and equations to represent mathematical relationships and solve real-world equations (DSP 5.2, 5.4)	• **Fluently multiply** multi-digit whole numbers using the standard algorithm • **Evaluate** variable expressions that involve a single operation • **Use** order of operations to **evaluate** single variable algebraic expressions with parentheses • **Explain** the difference between algebraic and arithmetic expressions • **Use** variables to represent quantities in expressions and number sentences • **Write** and **evaluate** algebraic expressions with two variables • **Use** a number line to compare and order integers • **Identify** the absolute value of an integer • **Identify** opposite integers • **Use** a model to add and subtract integers • **Identify, express** and **apply** the commutative and associative properties of whole numbers and identify properties of addition and multiplication • **Use** commutative and associative properties to solve problems, estimate, and compute • **Demonstrate** equivalence with the commutative, distributive and associative properties of whole numbers • **Demonstrate** the equivalence of both sides of an equation as the same value is added, subtracted, multiplied, or divided on each side • **Model** and **solve** one step equations using materials that model equivalence • **Represent** mathematical relationships using variables in expressions, equations and inequalities • **Describe** how a change in one variable relates to a change in a second variable in a practical situation • **Determine** the nature of changes in linear relationships using graphs, tables, and equations • **Use** a table to explore functions and graph them • **Add and subtract** fractions with unlike denominators by replacing given fractions with equivalent fractions to produce an equivalent sum or difference with like denominators • **Solve word problems** involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem • Use benchmark fractions and number sense of fractions to **estimate mentally and assess** the reasonableness of answers. *For example, recognize an*

Patterns **Ratios** **Decimals, Fractions & Percents** **Probability**	To describe features of a data set (DSP 5.1) To model, identify, compare, and relate rational numbers (NOA 5.2) To compare quantities and solve problems using ratios, rates and *percents*. (NOA 5.4) To determine the likelihood of certain events through games and simple experiments (DSP 5.3) To represent, extend and analyze numerical and geometric patterns (NOA 5.4)	*incorrect result* $^{2}/_{5} + ^{1}/_{2} = ^{3}/_{7}$, *by observing that* $^{3}/_{7} < \frac{1}{2}$ • **Interpret** a fraction as division of the numerator by the denominator ($^{a}/_{b} = a \div b$). • **Solve word problems** involving division of whole numbers leading to answers in the form of fractions or mixed numbers • **Calculate** the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. • **Multiply** fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas • **Compute** the mean of a set of data • **Use** range, mean, median, and mode to explain data • **Describe** how a change in an outlier can change the measures of central tendency • **Locate** points on a four quadrant coordinate grid by using ordered pairs • **Generate** a table of equal ratios and graph the ordered pairs • **Choose** and **use** benchmarks to **approximate** locations on number lines and coordinate grids • **Read, write,** and **illustrate** ratios using three standard forms • **Use** a table to generate equal ratios, w**rite** equal ratios, and **tell** if two ratios form a proportion • **Use** cross products, multiplication and division to **find** equivalent ratios • **Generate** a table of equal ratios and graph the ordered pairs • **Read** and **write** rates, and change a rate to a unit rate • **Illustrate and describe** the relationship between *decimals*, *fractions* and *percents* • **Represent** a rational number in its equivalent fraction, decimal, ratio and percent forms with models, number patterns and common factors • **Illustrate and describe** the relationship between *decimals*, *fractions* and *percents* • **Represent** a rational number in its equivalent fraction, decimal, ratio and percent forms with models, number patterns and common factors • **Estimate and find** *percents* using benchmarks and number pattern • **Find** what percent one number is of another • **Solve** problems involving sales tax and discounts • **Make** and **test** predictions of probability and fairness • **Design and conduct** probability experiments and games of chance

		• **Express** probability as a fraction • **Conduct** probability experiments and express the probability based on possible outcomes • **Identify** possible outcomes and **express** the likelihood of events as a fraction • **Make** generalizations about patterns and relationships and **test** those generalizations

G5:Q4

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Fractions	To solve real-world problems involving multiplication of fractions and mixed numbers (NOA 5.4)	• **Apply and extend** previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions a. **Interpret** division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $(1/3) \div 4$*, and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$. b. **Interpret** division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div (1/5)$*, and use a visual fraction model to show the quotient.* Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$. c. **Solve** real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many* $1/3$*-cup servings are in 2 cups of raisins?*
Integers		• Use positive and negative integers to **describe** quantities such as temperature above/below zero, elevation above/below sea level, or credit/debit.
Polygons **Measuring and Drawing Angles** **Parallel and Perpendicular Lines; Triangles** **Symmetry** **Perimeter and Area**	To describe and develop relationships between geometric properties of polygons and solids (G 5.1) To identify and generalize relationships between measurable attributes of plane and solid figures (G 5.1) To identify, draw and describe elements needed to explain spatial relationships (G 5.2) To identify and generalize relationships between measurable attributes of plane and solid figures (G 5.1) To identify, draw and describe elements needed to explain spatial relationships (G 5.2) To use coordinate systems to	• **Make** and **test** conjectures about geometric relationships • **Identify, describe, classify** and **draw** polygons • **Identify, compare and contrast** regular and irregular polygons • **Use** a protractor to measure angles • **Use** angles **to measure** and **classify** polygons • **Use** geometric relationships such as parallel, perpendicular, similar and congruent to describe the attributes of sets and subsets of shapes and solids • **Identify, describe** and **classify** triangles according to sides and angles • **Develop** and **apply** the formulas for perimeter and area of triangles • **Identify** line and rotational symmetry • **Demonstrate and describe** the relationship between area and perimeter when the dimensions of a polygon change • **Apply** formulas to find the perimeter and area of squares and rectangles • **Develop** and **apply** the formulas for perimeter and area of triangles • **Describe** relationships between the lengths of sides of rectangles and their areas and perimeters and

	identify and illustrate spatial location and geometric relationships (G 5.2)	generalize the patterns as simple formulas • **Find** strategies for estimating and measuring the perimeters and areas of irregular shapes • **Identify** and **measure** the parts of a circle (radius, diameter, chord, central angle) • **Identify** the meaning of pi • **Find** the circumference of a circle using a formula • **Find** the area of a circle • **Develop** strategies to determine the formula for the volume of rectangular solids • **Identify** line and rotational symmetry • Identify translations, rotations, and reflections • **Explain** the results of dividing, combining, and transforming shapes and the effects of slides, flips, and turns • **Draw** and **interpret** simple maps using coordinate systems and shapes or pictures • **Plot** points on the rectangular coordinate system and **estimate** and **determine** the distance between points

Resources for the Grade Five Math Literacy Connections	
Strand	***Book Titles***
Number Theory	*Is a Blue Whale the Biggest Thing There Is?* By Robert Wells, Whitman & Company, 1993 *Fractions*, by David Steinecker. Benchmark Books, 1996 *Locks, Crocs & Skeeters*, by Nancy Winslow Parker. Greenwillow Books, 1996 *Accidents May Happen*, by Charlotte Fultz Jones. Delacorte Press, 1996 *The Librarian Who Measured the Earth*, by Kathryn Lasky. Little, Brown & Co., 1994
Algebra	Logical reasoning puzzle books
Geometry	Pentominoes, Tangrams, Geoboards
Whole Numbers	*Let's Investigate Estimating* by Marion Smoothey. Marshall Canvendish Corporation, 1995 *Larson Leapfrog Math*, Meridian Creative Group (software)
Measurements	*Spaghetti and Meatballs for All!* By Marilyn Burns. Scholastic, 1997. Geoboards
Probability, Statistics & Graph	Microsoft Works or Excel (graph survey results)

Suggested Cross Curricular and Catholic Social Teaching Links

Grade Five

- Students create equations based on the calories found in different kinds of food and create menus that are nutritious. (Math, Health)
- Students will create and measure the effects of plans to conserve energy, reflecting an understanding of the call to be stewards of this earth. (Science, Math, Religion)

GRADE 6 ADH STANDARDS

Number Theory & Algebraic Concepts (NAC)	
NA.6.1	Understand meanings of operations and how they relate to one another to solve real world problems
NA.6.2	Compute fluently with multi-digit numbers and find common factors and multiples
NA.6.3	Represent and analyze quantitative relationships in a variety of ways to solve problems
NA.6.4	Analyze patterns, relations, functions, and change in various contexts
NA.6.5	Represent and analyze mathematical situations and structures using algebraic symbols to determine equivalence and solve problems
NA.6.6	Understand ratio concepts and use ratio reasoning to solve problems.
NA.6.7	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
NA.6.8	Apply and extend previous understandings of numbers to the system of rational numbers.
NA.6.9	Reason about and solve one-variable equations and inequalities.
NA.6.10	Represent and analyze quantitative relationships between dependent and independent variables.
Geometry (G)	
G.6.1	Solve real-world and mathematical problems involving area, surface area, and volume.
G.6.2	Specify locations and describe spatial relationships using coordinate geometry and other representational systems
G.6.3	Use visualization, spatial reasoning, and geometric modeling to solve problems
Measurement (M)	
M.6.1	Apply appropriate techniques, tools and formulas to determine measurements to solve real world problems
Data Analysis, Statistics, & Probability (DSP)	
DSP.6.1	Formulate questions that can be addressed with data; collect, organize, and display relevant data to answer them using appropriate statistical and graphical methods
DSP.6.2	Analyze data sets to form hypotheses and make predictions
DSP.6.3	Develop understanding of statistical variability
DSP.6.4	Summarize and describe distributions.

GRADE 6 KEY FLUENCIES:

- **COMPLETE UNDERSTANDING OF MULITPLICATION & DIVISION FLUENCY**
- **COMPLETE UNDERSTANDING OF ADDITION, SUBTRACTION, MULTIPLICATION, & DIVISION OF FRACTIONS FLUENCY**
- **CONNECT RATIO & RATE TO WHOLE NUMBER MULTIPLICATION & DIVISION**
- **EXTEND THE NOTION OF NUMBER TO THE SYSTEM OF RATIONAL NUMBERS TO INCLUDE NEGATIVE NUMBERS**
- **WRITING, INTERPRETING, AND USING EXPRESSIONS AND EQUATIONS**
- **DEVELOP UNDERSTANDING OF STATISTICAL THINKING**

GRADE 6 – Essential Understandings	Guided Questions -- What Students Need to Know
Mathematics can be used to describe, understand, and communicate about the world in order to solve problems and make decisions. Characteristics of a situation or problem influence the choice of numbers, operations, strategies, and tools. Patterns aid description, understanding, and communication about the world. Patterns and number relationships can be used to investigate, understand, and solve problems. Geometry has many real-world applications including design, architecture, and art. Measurement allows description, understanding, and communication about the world. Data collection and analysis can be used to predict outcomes, solve problems, and make decisions.	What does mathematics reveal about the world? What situations require the use of mathematical understandings? How do the characteristics of a situation influence the choice of numbers, operations, strategies, and tools? How is a solution determined to be reasonable, accurate, and complete? How and why are patterns used and where can they be found in human-designed environments? How are patterns and number relationships represented symbolically (such as consecutive odd numbers)? How are tables, graphs, and equations used to represent, analyze, and extend patterns? How are patterns used to solve problems and communicate information? What kinds of strategies help reveal patterns and number relationships? How does the precision required for a measurement influence the choice of strategies and tools? How is the understanding and communication about measurement used to solve problems and make decisions? What factors influence the way data is collected and organized? How is the analysis of data used to solve problems? How is the reliability of data affected by the source, quantity, and method of collection? How is the presentation of data used or misused to support different points of view?

G6:Q1

TOPICS	OBJECTIVES	ENABLING OUTCOMES; The students will…
Whole Numbers **Estimation**	To compute fluently with multi-digit numbers (NAC 6.1) To represent numbers in expanded and regrouped forms in the base ten place value system (NAC 6.3) To extend whole number place value concepts to include decimal numbers which are also represented as *fractions* whose denominators are multiples of ten (NAC 6.8) To use place value concepts, number patterns and properties to develop and apply estimation	• **Fluently divide multi-digit numbers using the standard algorithm.** • **Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation** • **Locate, order** and **compare** whole numbers on number lines, scales and the coordinate grid • **Compare** large numbers using expanded forms and powers of ten • **Read, write, count, skip count, order, compare**, **round**, and **expand** numerals to one billion • **Identify** negative exponents by examining patterns **Write** expanded numerals in standard form • **Express** a standard form number in scientific notation and vice versa • **Read** and **write** *decimals* to ten thousandths place in standard form as number words • **Round** *decimals* to the nearest ten thousandths place • **Compare and order** decimals

Whole Numbers and Decimals **Number Theory** **Fractions**	and computation strategies (NAC 6.4) To apply place value concepts and number properties to the addition, subtraction, multiplication and division of multi-digit numbers (NAC 6.2) To use factors to explore, represent and classify numbers (NAC 6.2) To model , identify and express equivalent forms of numbers expressed as whole numbers, *fractions* and mixed numbers (NAC 6.8) To use models, number lines, scales and a coordinate grid to represent and illustrate decimal numbers and to express them in equivalent forms (NAC 6.3) To add and subtract *fractions* and mixed numbers using models, pictures and number sentences (NAC 6.7) To apply and extend previous understandings of multiplication and division to divide fractions by fractions (NAC 6.3) To use models and pictorial representations to develop concepts and methods by which	• **Convert** decimals to fractions • **Estimate** and **predict** reasonable answers and **recognize** and explain when an estimate will be more or less than an exact answer • **Explain** orally and in writing when a situation requires an exact answer or when an estimate is sufficient • **Develop**, **describe**, and **use** a variety of ways to estimate and calculate with large numbers and connect the strategies to powers of ten • **Use** benchmarks to understand the relative magnitude of numbers • **Use** place value concepts, number patterns, the number line and the commutative, associative, and distributive properties to **develop** estimation and computation strategies • **Select** and **apply** the most suitable estimation strategy: rounding, clustering, front end (with adjustment), compatible numbers, compensation • **Recognize** place value patterns when multiplying and dividing *decimals* by powers of 10 • **Use** the distributive property [10 x (4+5) = (10 x 5) + (10 x 4)] to estimate, multiply and divide multi-digit numbers by one-digit factors • **Identify** and **use** the inverse relationships of multiplication and division to solve and check problems **Determine** the proper operation to solve a problem and **justify** the reasoning • **Locate, order** and **compare** *decimals* on number lines, scales and the coordinate grid **Multiply** and **divide** *decimals* by *decimals* • **Determine and write** the prime factorization of any whole number • **Represent** numbers by using exponents • **Change** exponent form to standard numeral, write as repeated factors and vice versa • **Use** factors of composite numbers, powers of ten and divisibility rules to **find** products and missing factors • **Memorize** and **apply** the divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10 • **Explain** orally and in writing when a situation requires an exact answer or when an estimate is sufficient • **Locate, order** and **compare** *fractions* on number lines, scales and the coordinate grid • **Determine** the decimal equivalents of *fractions* • **Convert** *fractions* to *decimals*, *decimals* to *fractions*, and *fractions* to *percents* • **Change** a fraction to a decimal using division • **Write** *fractions* as terminating and repeating *decimals* • **Convert** repeating *decimals* to *fractions*

	to multiply and divide *fractions* and mixed numbers (NAC 6.3)	• **Interpret and compute** quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem • **Add and subtract** *fractions*, whole numbers and mixed numbers using a variety of computational strategies • **Subtract** mixed numbers with renaming • **Identify** reciprocal numbers • **Apply** reciprocal numbers to division of a whole number by a fraction • **Use** models to **divide** whole numbers by *fractions* and *fractions* by whole numbers • **Multiply** and **divide** *fractions*, whole numbers and mixed numbers using a variety of computational strategies • **Use** cancellation in multiplication of *fractions* • **Model** and **describe** when products or quotients with *fractions* and *decimals* can yield a larger or smaller result than either factor • **Write** whole number division problems in fraction form and round the fraction form to estimate an answer to a division problem • **Write** division problems in fraction form • **Express** remainders in division as *fractions*

G6:Q2

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Integers **Functions** **Rational Numbers**	To explore numbers less than zero and extend the number line to illustrate concepts and computation strategies of integers (NAC 6.8) To use factors to explore, represent and classify numbers (NAC 6.2) To recognize and demonstrate equivalence using number properties (NAC 6.1) To recognize, use , simplify and evaluate arithmetic and algebraic expressions (NAC 6.5) To write and analyze expressions, equations and inequalities that express relationships between numbers (NAC 6.9) To recognize, use , simplify and evaluate arithmetic and algebraic expressions (NAC 6.9)	• **Define** and **recognize** integers • **Understand** that positive and negative numbers are used together to describe quantities having opposite directions or values; **represent** quantities in real world context • **Use** a number line to illustrate, compare and order integers • **Identify and demonstrate** the absolute value of an integer as its distance from 0 on the number line • **Demonstrate understanding** of ordering and absolute value of rational numbers • **Write, interpret, and explain** statements of order for rational numbers in real world contexts • **Identify** opposite integers • **Add, subtract**, **multiply and divide** integers • **Memorize** and **apply** the rules for the order of operations including parentheses and exponents • **Apply** the **complete order** of operations in evaluating expressions • **Identify, express** and **apply** the commutative, distributive, and associative properties of whole numbers • **Use** order of operations to **evaluate** expressions including exponents • **Contrast** constants and variables • **Evaluate** algebraic expressions and formulas • **Demonstrate** how to maintain equivalence in equations • **Model** and **solve** one step linear equations by maintaining equivalence (use inverse operations) • **Represent** mathematical relationships using variables in expressions, equations and inequalities • **Describe** how a change in one variable relates to a change in a second variable in a practical situation • **Represent** numerical and contextual situations with algebraic expressions, equations and inequalities • **Use** variables as placeholders, to denote a pattern, to write a formula and to represent a function or relation • **Write** and **evaluate** algebraic expressions with two variables • **Recognize, identify, and apply** the inverse property of addition and multiplication • **Recognize, identify, and apply** the addition, subtraction, multiplication, and division properties of equality • **Recognize, identify, and apply** the identity properties of addition and multiplication • **Identify and apply** the distributive property of addition and multiplication

G6:Q3

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Functions **Patterns** **Ratios** **Probability** **Plane and Solid Figures** **Measurable Attributes**	To represent numerical and linear relationships in graphic forms (NAC 6.10) To represent, extend and analyze numerical and geometric patterns (NAC 6.4) To demonstrate and apply knowledge of patterns, ratios, and proportions. (NAC 6.4) To develop an understanding of probability and write probability as a fraction, decimal, or percent. Students will describe and develop relationships between geometric properties of plane and solid figures. (G 6.2) To identify and generalize relationships between measurable attributes of plane and solid figures (M 6.1) To identify, draw and describe elements needed to explain spatial relationships (G 6.1, 6.3)	• **Choose** and **use** benchmarks to **approximate** locations on number lines and coordinate grids • **Locate** points on a four quadrant coordinate grid by using ordered pairs • **Use** a table to explore functions and graph them • **Determine** the nature of changes in linear relationships using graphs, tables, and equations • **Apply** the addition, subtraction, multiplication, and division properties of equality to solve and check one-step algebraic equations • **Apply** the addition, subtraction, multiplication, and division properties of equality to solve and check one-step algebraic equations ($2x = 4$; $x + 5 = 8$) • **Solve** real-world and mathematical problems by **writing and solving** equations • **Recognize** that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions • **Represent** solutions of inequalities on number line diagrams • **Represent and analyze** quantitative relationships between dependent and independent variables • **Identify and express ratios using appropriate notation (i.e., a/b, a to b, a:b)** • **Identify and explain equivalent ratios** • **Explain ratios that represent a real world situation** • **Design tables of equivalent ratios relating quantities and use tables to compare ratios** • **Solve problems involving the probability of a simple event, including the probability as a fraction, decimal, or percent** • **Describe, analyze** and **extend** numeric, geometric and statistical patterns • **Make** generalizations about patterns and relationships and **test** those generalizations • **Extend** and **compare** arithmetic and geometric sequences • **Represent** geometric and numeric patterns using words, tables, graphs and equations • **Analyze** patterns and data to make predictions • **Determine** the nature of changes in linear relationships using graphs, tables, and equations

Spatial Relationships		• **Make** and **test** conjectures about geometric relationships • **Classify** polygons according to their transformational properties • **Use** the relationships of sides and angles to **classify** sets of polygons • **Make** and **test** conjectures about side and angle relationships and congruence • **Identify, compare and contrast** regular and irregular polygons • **Use** angles to **measure** and **classify** polygons • **Identify** and **classify** angles as complementary and supplementary • **Use** a protractor to measure angles • **Use** the rectangle as a basic shape to model and develop formulas for the area of triangles, parallelograms, trapezoids and circles • **Use** a compass to **draw** a circle • **Find** the area of a circle • **Find** the circumference of a circle using a formula • **Identify** and **measure** the parts of a circle (radius, diameter, chord, central angle) • **Describe** the relationships between and among radius, diameter, circumference and area of a circle • **Identify** the meaning and value of pi • **Determine** the volume of rectangular solids • **Describe** the relationships between the measures of area of two-dimensional objects and volume of three dimensional objects • **Develop and use** formulas to determine the volume of pyramids and cylinders • **Calculate** the surface area of a rectangular prism • **Represent** the surface of three-dimensional objects through the use of two-dimensional nets • **Identify** rotational symmetry and points of rotation • **Use** spatial reasoning location and geometric relationships to solve problems

G6:Q4

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Ratios, Rates, Percents	To compare quantities and solve problems using ratios, rates and *percents* (NAC 6.6)	• **Use** cross products, multiplication and division to find equivalent ratios **interpreting** maps and scale drawings or **identifying** probability • **Read** and **write** rates, and change a rate to a unit rate • **Convert** between rates using ratios and proportions • **Memorize** common percent-fraction equivalents (benchmarks) • **Find** the percent of a number • **Find** what a percent one number is of another • **Write** percents greater than 100% and less than 1% as decimals and fractions • **Generate** a table of equal ratios and graph the ordered pairs • **Solve** problems involving sales tax and discounts
Measurement	To coordinate systems to identify and illustrate spatial location and geometric relationships (M 6.1)	• **Use** different ratios to convert between units of length, area, and volume in the customary and metric systems • **Recognize** and **use** powers of ten as conversion ratios in the metric system • **Compute** customary and metric measurements with regrouping recording answer in simplified form • **Select, justify, convert, metric and standard** units of measurement • **Explain** the difference between mass and weight
Graphs	To collect, organize, describe, and apply data (DSP 6.1)	• **Use, read, create, interpret,** and **compare** a variety of graphic organizers, charts, and graphs • (These charts, graphs, etc. should include Venn diagrams, histograms, broken line graphs, bar graphs, picture graphs, circle graphs, stem and leaf, and scatter plots.)
Probability	To determine the possible outcomes and likelihood of certain events through games and simple experiments (DSP 6.2)	• **Conduct** probability experiments and express the probability based on possible outcomes • **Design** and **conduct** probability experiments and make predictions about outcomes that are equally likely or not equally likely • **Relate** the likelihood of an event to a numerical value • **Identify** possible outcomes and express the likelihood of events as a fraction • **Explain** that probabilities are more reliable to use as predictors when there is a large number of trials • **Describe** the relationship between the number of trials in an experiment and the predicted outcomes • **Express** probabilities as *fractions*, ratios, *decimals* and *percents*
Data Analysis	To pose questions to be answered through collection and analysis of a data set (DSP 6.1, 6.2) To describe and analyze features of a data set (DSP 6.3, 6.4)	• **Use** a variety of ways to **collect, organize**, **record, analyze**, and **interpret** data and identify patterns and

	To review and summarize critical fraction concepts: simplify, rename, convert mixed to improper to mixed, add, subtract, multiply, and problem solving ***Review and summarize critical understanding of the relationship between fraction, decimal, and percents***	trends • **Use** technology to **create** spreadsheets and **convert** information into graphs • **Use** extended numeric, geometric and statistical patterns to identify trends and justify predictions • **Differentiate** between numerical and categorical data and their appropriate representations • **Analyze** patterns and data to make generalizations and predictions • **Describe** the shape of data sets using measures of spread (range and outliers) and central tendency (mode, median, and mean) • **Recognize** that changes in a data set can affect the mode, median, mean, and range • **Recognize** misleading data • **Solve problems involving addition and subtraction of fractions and mixed numbers, and express answers in simplest form** • **Make estimates appropriate to a given situation** • **Analyze what effect the estimation method used has on the accuracy of results**

Suggested Cross Curricular and Catholic Social Teaching Links **Grade Six**
• Students read *From the Mixed-Up Files of Mrs. Basil E. Frankweiler* and create proportions that measure the difference in the cost of subway fare, food, etc. described in the book with current day costs and make generalizations about the increase in the cost of living from the 1960's to the present. Online resources should be used. (Math, Language Arts) • Students create a budge for a service project, such as providing a meal for a local soup kitchen. (Religion, Math)

STANDARD GRADE 7 *TRANSITIONAL* CURRICULUM

Number and Operations – Grade Seven	
Essential Understandings	**Guided Questions -- What Students Need to Know**
Mathematics can be used to describe, understand, and communicate about the world in order to solve problems and make decisions. Characteristics of a situation or problem influence the choice of numbers, operations, strategies, and tools.	What does mathematics reveal about the world? What situations require the use of mathematical understandings? How does mathematics enable people to work with intangible phenomena (such as distance, space, and nanosecond)? How do concrete materials model mathematical situations? How do the characteristics of a situation influence the choice of operations, strategies, and tools? How is a solution determined to be reasonable, accurate, and complete?

TOPICS	ENABLING OUTCOMES
	Students will:
Integers	• identify, order, and compare integers • graph integers on a number line • **add, subtract, multiply, and divide integers and explain their operational processes**
Rational numbers	• identify, order, and compare rational numbers • graph rational numbers on a number line • **apply properties of operations as strategies to add, subtract, multiply, and divide rational numbers and explain their operational processes** • describe situations in which opposite quantities combine to make 0 • **understand subtraction of rational numbers as adding the additive inverse** • **convert rational numbers to decimals and classify as terminating, non-terminating, and repeating** • **solve real-world and mathematical problems involving the four operations of rational numbers**
Real numbers	• **classify real numbers as rational, irrational, whole, integer, or natural**
Percents	• convert between decimal, fraction, and percent formats • **compare and order percents (including those less than one and greater than 100)** • calculate the percent of a number (20% of 50) including applications to o **tax and discount** o simple interest o commissions o gratuities o percent of change
Ratios	• recognize and represent proportional relationships between quantities • identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships • solve ratio equations **using cross-multiplication** • solve word problems involving **ratios and proportions, including the percent proportion** (16 is what percent of 90) • **apply ratios and solve problems involving scale, models, and unit rates**
Exponents and roots	• calculate perfect square roots • **estimate the value of a non-perfect square root to a given decimal point value**

Geometry and Measurement – Grade Seven

Essential Understandings	Guided Questions -- What Students Need to Know
Attributes and relationships of plane and solid figures, objects, and patterns can be used to describe, understand, and communicate about the world. Geometry has many real-world applications including design, architecture, and art. Measurement allows description, understanding, and communication about the world.	How can geometry be seen in the natural and human-designed environments? How are distance, direction, coordinates, and scale used to understand and explain the arrangement of objects and locations? How do models and scale drawings enhance understanding used in problem-solving and design? How do the characteristics of geometric shapes and figures influence their use in aesthetic and functional designs? How is measurement used to quantify information about objects and events? How do the characteristics of objects and events influence the choice of measurement strategies and tools? How does the precision required for a measurement influence the choice of strategies and tools? How is the understanding and communication about measurement used to solve problems and make decisions?

TOPICS	ENABLING OUTCOMES
	Students will:
Plane figures	• prove the similarity of plane figures **by identifying congruent angles and proportional sides** • solve problems involving scale drawings • calculate the lengths of sides of similar plane figures • **sketch, draw, and construct geometric shapes with given conditions using ruler, protractor, compass, and technology** • **construct triangles from three measures of angles or sides** • **verify the properties of dilations, rotations, reflections, and translations and use these properties to compare two-dimensional figures**
Solid figures	• **describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids**
Formulas	• **develop and/or use formulas** to calculate surface area and volume for solid figures (cone, sphere, pyramid, prism, cylinders) • **develop and/or use formulas to calculate the area and circumference of circles** • **develop and/or use formulas** to calculate the area and perimeter of plane figures

Algebra – Grade Seven	
Essential Understandings	**Guided Questions -- What Students Need to Know**
Patterns aid description, understanding, and communication about the world. Patterns and number relationships can be used to investigate, understand, and solve problems.	How and why are patterns used and where can they be found in human-designed environments? How are patterns and number relationships represented symbolically (such as consecutive odd numbers)? How are tables, graphs, and equations used to represent, analyze, and extend patterns? How are patterns used to solve problems and communicate information? What kinds of strategies help to reveal patterns and number relationships?

TOPICS	ENABLING OUTCOMES
Expressions	Students will: • **apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients** • **translate an expression from written to algebraic form and from algebraic to written form** • **identify and combine like terms** (2x + 3x = 5x)
One-variable linear equations and inequalities	• solve and check **two-step equations** (2x + 3 = 5) **using rational numbers and the distributive property** [2 (x + 3) = 8] • **solve, check, and graph the solution to one- and two-step one-variable linear inequalities excluding multiplication or division by a negative** [2x > 8; x – 5 < -9] • solve **multi-step** real-life mathematical problems posed with positive and negative rational numbers in any form by constructing simple equations and inequalities • evaluate solutions for reasonableness, accuracy, and completeness

Data Analysis and Probability – Grade Seven	
Essential Understandings	**Guided Questions -- What Students Need to Know**
Data collection and analysis can be used to predict outcomes, solve problems, and make decisions. Probability supports making predictions, drawing conclusions, and solving problems.	What factors influence the way data is collected and organized? How is the analysis of data used to solve problems? How is the reliability of data affected by the source, quantity, and method of collection? How is the presentation of data used or misused to support different points of view? How are the probability and odds of an event determined and expressed? What factors influence the certainty and uncertainty of an event? How is probability used to make predictions and draw conclusions?

TOPICS	ENABLING OUTCOMES
Probability and statistics	Students will: • differentiate between theoretical and experimental probability • investigate chance processes and develop, use, and evaluate probability models • calculate and interpret the probability of simple events • understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • find probabilities of compound events using organized lists, tables, tree diagrams, and simulation
Graphs	• **predict and infer** data from a variety of graphs • use random sampling to draw inferences about a population • draw informal comparative inferences about two populations

PREALGEBRA ADH STANDARDS for ACCELERATED 7TH GRADE OR STANDARD 8TH GRADE

The Number Theories (NT)

PA.NT 7.1	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
PA.NT 7.2	Compute flexibly and fluently and make reasonable estimates; APPLY TO REAL WORLD SITUATIONS
PA.NT 7.3	Understand and describe patterns and functional relationships

Expressions and Equations (EE)

PA.EE 7.1	Use properties of operations to generate equivalent expressions.
PA.EE 7.2	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
PA.EE 7.3	Use mathematical models to represent and understand quantitative relationships

Ratios and Proportional Relationships (RP)

PA.RP 7.1	Analyze proportional relationships and use them to solve real-world and mathematical problems

Geometry & Measurement (GM)

PA.GM 7.1	Analyze characteristics and properties of two and three dimensional geometric shapes and develop mathematical arguments about relationships
PA.GM 7.2	Use properties and characteristics of two-and three-dimensional shapes and geometric theorems to describe relationships, communicate ideas and solve problems
PA.GM 7.3	Develop and apply appropriate techniques, tools and formulas to estimate and determine measurements

Data Analysis, Statistics, and Probability (DSP)

PA.DSP 7.1	Collect, organize and display data using appropriate statistical and graphical methods.
PA.DSP 7.2	Select and use appropriate statistical methods to analyze data
PA.DSP 7.3	Analyze data sets to form hypotheses and make predictions
PA.DSP 7.4	Investigate chance processes and develop, use, and evaluate probability models

Pre-Algebra GRADE KEY FLUENCIES:

Extend understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems.

Develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers.

Developing understanding of operations with rational numbers and working with expressions and linear equations.

PA:Q1

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Place Value **Scientific Notation** **Measure of Central Tendency** **Stem-leaf & Box, Whisker Plots** **Estimation**	To represent numbers in expanded and regrouped forms in the base ten place value system (NT 7.3) To use powers of ten and scientific notation to write very large and very small numbers (NT 7.3) To understand and apply number concepts appropriately and accurately (NT 7.2) To describe and analyze features of a data set and justify conclusions (DSP 7.1, 7.3) To collect and construct appropriate representations of data (DSP 7.2)) To use place value concepts, number patterns and properties to develop and apply estimation and computation strategies to include negative numbers (NT 7.1, 7.2)	• **Locate, order** and **compare** whole numbers on number lines, scales and the coordinate grid • **Graph** integers on a number line • **Add, subtract, multiply, and divide** integers and explain their operational success • **Compare** large numbers using expanded forms and powers of ten • **Write** expanded numerals in standard form • **Express** a standard form number in scientific notation and vice versa • **Identify** negative exponents by examining patterns • **Use** powers of ten and positive exponents to express and compare magnitude of very large numbers and connect to scientific notation • **Develop, describe** and **use** a variety of methods to estimate and calculate with very large numbers • **Use** powers of ten and negative exponents to write *decimals* as *fractions* • **Use** powers of ten and positive and negative exponents **to express** and **compare** magnitude of very large and very small numbers and **connect** to scientific notation • **Find** the results of multiplication and division with powers of ten using patterns in operating with exponents • **Determine** the prime factorization of any whole number • **Determine** the greatest common factor and least common multiple using proem factorization • **Find, use** and **interpret** measures of central tendency and spread, including mode, median, mean, range, and outliers • **Recognize** that changes in a data set can affect the mode, median, mean, and range • **Compare** two sets of data based on their distributions and measures of central tendency • **Analyze** and **interpret** data using descriptive statistics, including range, mode, median, quartiles, outliers, and mean • **Make** predictions from scatter plots using or estimating a line-of-best-fit • **Collect , organize, display, compare,** and **analyze** large data sets • **Construct** a variety of data displays including box and whisker plots • **Identify** where measures of central tendency and dispersion are found in graphical displays • **Develop, describe, and use** a variety of ways to estimate and calculate with very large and very small numbers and **connect** the strategies to powers of ten • **Use** place value concepts, number patterns, the number

		line and the commutative, associative, and distributive properties to **develop** estimation and computation strategies **Estimate to predict outcomes and determine reasonableness of results and to describe whether an estimate is an over- or underestimate**

PA:Q2

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Rational Numbers	To identify, compare, and relate rational numbers (EE 7.1) To use factors to explore, represent and classify numbers (EE 7.2) To represent practical situations and solutions to problems using the appropriate symbolic form –*fractions*, *decimals*, or *percents* (EE7.2) To identify relationships that are linear and nonlinear and compare and contrast their properties using tables, graphs, equations and verbal descriptions (EE 7.3)	• **Rewrite** rational numbers in equivalent fraction, decimal, ratio, and percent forms with number patterns and common factors • **Convert** rational numbers to decimals and classify as terminating. non-terminating, or repeating • **Classify** numbers in the real number system (counting, whole, integer, rational, and irrational) • **Identify** a rational number between any two rational numbers • **Find** absolute values of rational numbers • **Simplify** rational expressions • **Multiply** and **divide** rational expressions • **Add** and **subtract** rational expressions with like and unlike denominators • **Compare, locate, label, and order** rational numbers on number lines, scales, coordinate grids and measurement • **Graph** rational numbers on a number line • **Apply** properties of operation as strategies to add, subtract, multiply, and divide rational numbers and **explain** their operational processes • **Solve** real-world and mathematical problems involving the four operations of rational numbers
Prime Factorization		• **Find** prime factors and **write** prime factorization of numbers • **Represen**t numbers by using exponents • **Change** exponent form to standard numeral, write as repeated factors and vice versa • **Find** prime factorizations of integers and monomials • **Find** GCF of integers and monomials
Decimals	To apply place value concepts and properties of numbers to the addition, subtraction, multiplication and division of multi-digit integers (NT 7.1) To recognize and demonstrate equivalence using number properties (EE 7.1, 7.2)	• **Write** *percents* greater than 100% and less than 1% • as *decimals* and *fractions* • **Write** *fractions* as terminating and repeating *decimals* and vice versa • **Estimate and compute** with *fractions*, *decimals*, mixed numbers, improper *fractions*, ratios, proportions, and *percents*
Integers	To write and analyze expressions, equations and inequalities that express relationships between numbers (EE 7.2, 7.3)	• **Identify** the characteristics of functions and relations, including domain and range • **Determine** whether a relation is a function

Solving Equations **Solving 2-Step Equations** **Slope**	To use numbers, symbols, and words to represent and describe mathematical relationships (EE 7.3) To identify relationships that are linear and nonlinear and compare and contrast their properties using tables, graphs, equations and verbal descriptions (EE 7.3) To solve problems using a variety of algebraic representations (EE 7.2) To recognize and demonstrate equivalence using number properties (EE 7.1) To use numbers, symbols, and words to represent and describe mathematical relationships (EE 7.1 – 7.3) To identify relationships that are linear and nonlinear and compare and contrast their properties using tables, graphs, equations and verbal descriptions (EE 7.3)	• **Use** tables and graphs to measure and describe changes • **Graph** linear equations on an xy-axis • **Graph** functions from ordered pairs • **Find** function values • **Solve** problems with positive and negative numbers using models and number lines • **Add, subtract, multiply** and **divide** integers • **Use** order of operations including exponents • **Use** the order of operations to compute and solve a variety of multi-step problems, including those with parentheses and exponents • **Use** absolute value in solving problems • **Simplify** algebraic expressions by combining like terms • **Demonstrate** how to maintain equivalence in equations • **Model** and **solve** one step linear equations by maintaining equivalence (use inverse operations) • **Represent** numerical and contextual situations with algebraic expressions, equations, and inequalities **(Use** variables in patterns, formulas, functions and relations) • **Contrast** constants and variables • **Simplify** expressions that contain rational numbers • **Write** verbal expressions as algebraic expressions and sentences as equations • **Evaluate** expressions with exponents • **Write** an equation given some of the solutions • **Evaluate** expressions with square roots • **Generalize** mathematical situations using variables in expressions, equations and inequalities • **Identify, express** and **apply** the commutative, distributive, and associative properties of whole numbers • **Use** functional notation **to express** algebraic relationships • **Graphically** find the solution to a system of equations • **Represent** numerical and contextual situations with algebraic expressions, equations and inequalities • **Evaluate** algebraic expressions and formulas • **Solve** problems using concrete, verbal, symbolic, graphic and tabular representations • **Solve** equations in one variable that contain absolute value expressions • **Model** and **solve** two-step linear equations using a variety of methods (i.e., concrete materials, algebra tiles, pictorial representations, etc.) • **Graph** inequalities on the coordinate plane • **Recognize** that a linear relationship has a constant rate of change called slope • **Find** the slope of a line

		• **Use** graphs, tables, equations and verbal descriptions to represent and analyze changes in linear and nonlinear relationships • **Identify** the *x* and *y* intercepts • **Describe** what a line will look like before it is graphed, i.e. if the line is in a positive or negative direction, and how steep the line should be by analyzing the slope • **Solve** linear equations for "y" given the linear equation in any other form • **Determine** the solutions of linear equations (0, 1, or an infinite number) • **Identify** and **write** the equation for a line in point-slope, slope-intercept and standard forms

PA:Q3

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Fractions **Ratios and Rates** **Solve Proportions** **Work with Percents** **Theoretical and Experimental Probability** **Data Analysis**	To represent practical situations and solutions to problems using the appropriate symbolic form –*fractions*, *decimals*, or *percents* (RP 7.1) To determine probabilities and outcomes (DSP 7.4) To describe and analyze features of a data set and justify conclusions (DSP 7.1-7.4)	• **Convert** *fractions* to *decimals*, *decimals* to *fractions*, *fractions* to *percents*, and *percents* to *fractions* (including repeating *decimals*) • **Write** *fractions* as terminating and repeating *decimals* and vice versa • **Use** the distributive property to multiply and divide mixed numbers and *decimals* • **Estimate and compute** with *fractions*, *decimals*, mixed numbers, improper *fractions*, ratios, proportions, and *percents* • **Compare** and **order** percents • **Write** and **use** ratios, rates, and unit rates • **Write and solve** proportions • **Use** proportions to solve problems involving geometric figures • **Use** proportions and similar figures to measure objects indirectly • **Calculate** the percent of a number (i.e., 20% of 50) including applications to: **a.** Tax and discount **b.** Simple interest **c.** Commissions **d.** Gratuities **e.** Percent of change • **Estimate and use** common applications of *percents* • **Estimate** and **solve** problems involving percent of increase and decrease • **Identify** experimental probability by gathering data from experiments • **Identify** theoretical probability by analyzing possible and likely outcomes • **Conduct** experiments and compare experimental to theoretical probabilities • **Solve** problems involving the probability of simple and

Graphs and Data Analysis	To analyze physical phenomena and patterns to identify relationships and make generalizations (NT 7.3) To collect and construct appropriate representations of data (DSP 7.2)	compound events in familiar contexts • **Make** and **evaluate** statistical claims and **justify** conclusions with evidence • **Identify** trends and **justify** conclusions • **Describe** the role of random sampling, random number generation, and the effects of sample size on statistical claims • **Distinguish** between combinations and permutations as ways to predict possible outcomes in certain situations • **Use** combinations and permutations, trees, and networks (counting strategies) in a variety of contexts • **Identify** when order is irrelevant in determining a solution • **Determine** the nature of changes in linear relationships using graphs, tables, and equations • **Describe** in context how a change in one variable relates to a change in a second variable • **Identify** the independent and dependent variables in a given situation • **Formulate** questions, **design** surveys and samplings • **Organize** and **analyze** gathered data and **defend** the analysis • **Organize and display** data using graphical representations • **Make** and **defend** predictions based on patterns and trends • **Use** a matrix to organize and describe data

PA:Q4

TOPICS	OBJECTIVES	ENABLING OUTCOMES: The students will…
Geometry	To describe and develop relationships between geometric properties of plane and solid figures (GM 7.1) To identify and generalize relationships between measurable attributes of plane and solid figures (GM 7.1-7.3) To identify, draw, and describe elements needed to explain spatial relationships (GM 7.3)	• **Identify** which classes of polygons have line and/or rotational symmetry • **Identify** and **classify** angles as complementary or supplementary • **Develop and use** formulas to determine the volume of pyramids and cylinders • **Calculate** the surface area of a rectangular prism • **Describe** the effect of scale factors on the length, area, and volume ratios of similar polygons, circles, and solids • **Make** and **test** conjectures about the relationships among angles, sides, perimeters, and areas of congruent and similar polygons (Include the Pythagorean Theorem) • **Verify** the Pythagorean Theorem, using diagrams, concrete materials, and measurement • **Apply** the Pythagorean Theorem to find the missing length of a side of a right triangle when given the lengths of the other two sides • **Draw** and **interpret** nets, cross-sections, and front, side, and top views of various solids • **Use** rectangular grids to represent polygons and perform transformations (translations, rotations, reflections, and dilations) • **Describe** the effect of transformations on polygons with

Algebra	To solve problems using a variety of algebraic representations (EE 7.2) **To use tables, graphs, rules and words to investigate, describe, and analyze functional relationships in a variety of patterns (NT 7.3)**	line and/or rotational symmetry • **Construct** similar polygons on coordinate grids • **Describe** the similarity of polygons as a result of dilations (reductions or enlargements) and their effects on measurements • **Use** spatial reasoning, location, and geometric relationships to solve problems • **Apply** transformations (rotate or turn, reflect or flip, translate or slide, and dilate or scale) to geometric figures represented on a graph • **Identify** applications of transformations, such as tiling, fabric design, art, and scaling • **Develop** and **use** formulas to **determine** the surface area of three-dimensional objects • **Apply** properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • **Translate** an expression from written to algebraic form and from algebraic form to written form • **Add** and **subtract** polynomials • **Multiply** and **divide** monomials • **Multiply** a polynomial by a monomial • **Multiply** binomials • **Simplify** expressions involving powers of monomials and products and quotients of monomials • **Determine** the nature of changes in linear relationships using graphs, tables, and equations • **Describe, analyze,** and **extend** numeric, geometric and statistical patterns • **Make** generalizations about patterns and relationships and **test** those generalizations • **Represent, extend,** and **compare** geometric and numeric patterns using words, tables, graphs and equations • **Analyze** patterns and data to make predictions • **Write** recursive and explicit functions to **generalize** patterns • **Recognize** and **solve** problems of direct variation • **Solve and check** two-step equations ($2x + 3 = 5$) using rational numbers and the distributive property [$2(x + 3) = 8$] • **Solve, check, and graph** the solution to one and two-step one variable linear inequalities excluding multiplication or division by a negative [$2x > 8$; $x - 5 < -9$] • **Solve** multi-step real-life mathematical problems posed with positive and negative rational numbers in any form by consulting simple equations and inequalities

Suggested Cross Curricular and Catholic Social Teaching Links

Grade Seven/Eight

- Students write about and calculate the cost of war, natural disasters, unemployment, etc., expressing an understanding that, as Catholic Christians, we are called to work globally and locally for justice. (Math, Social Studies, Science)

- Students create graphs describing the inequality of the consumption of the world's resources and design service projects that address local and global injustice. (Math, Religion, Science)

ALGEBRA STANDARDS

A1. Understand and describe patterns and functional relationships
A2. Represent and analyze quantitative relationships in a variety of ways
A3. Use operations, properties and algebraic symbols to determine equivalence and solve problems
A4. Use properties and characteristics of two- and three-dimensional shapes and geometric theorems to describe relationships, communicate ideas, and solve problems
A5. Use spatial reasoning, location, and geometric relationships to solve problems
A6. Develop and apply units, systems, formulas and appropriate tools to estimate and measure

Algebra – Grade Eight	
Essential Understandings	**Guided Questions -- What Students Need to Know**
Mathematics can be used to describe, understand, and communicate about the world in order to solve problems and make decisions.	What does mathematics reveal about the world? What situations require the use of mathematical understandings? How does mathematics enable people to work with intangible phenomena (such as distance, space, and nanosecond)? How do concrete materials model mathematical situations?
Characteristics of a situation or problem influence the choice of numbers, operations, strategies, and tools.	How do the characteristics of a situation influence the choice of numbers, operations, strategies, and tools? How is it determined that a solution is reasonable, accurate, and complete?

Topics	Enabling Outcomes

Expressions	Students will:
	• interpret parts of an expression, such as terms, factors, and coefficients
	• make and justify predictions based on patterns
	• apply the appropriate properties of real numbers and the steps for order of operations to write, evaluate, simplify, add, subtract, multiply, and divide expressions:
	    ○ polynomial
	    ○ rational
	    ○ radical
	    ○ exponential including concept of scientific notation
	    ○ Introduce expressions containing zero and negative exponents (numbers, not variables)
	• derive the formula for the sum of a finite geometric series and use to solve problems
	
	• understand that a function, y = f(x), is a rule that assigns to each input (domain) exactly one output (range) - the graph of a function is the set of ordered pairs consisting of an input and the corresponding output
	    ○ determine whether a relation is a function
	    ○ compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal description)
	
	• use function notation to evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context

Equations, functions, and inequalities	• solve one-variable linear equations and inequalities
	    ○ interpret the solution to identify the number of acceptable solutions (e.g., zero, one, infinitely many solutions)
	    ○ write and use ratios, rates, and unit rates
	    ○ write and solve proportions, including percent problems
	    ○ solve, graph, and check the solution to any one-variable linear equation or inequality
	    ○ solve and graph the solution to compound linear equations and inequalities including absolute value ($x > 2$ and $x < 3$; $\|x\| = 3$)
	    ○ rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations (literal equations)
	
	• analyze and solve linear equations, functions, and pairs of linear equations and functions
	    ○ understand the connections between proportional relationships, lines, linear equations, and inequalities with relation to slope
	    ○ determine the constant rate of change in a linear relationship and recognize this as the slope of a line
	    ○ use slope as the change in "y" over the change in "x"
	    ○ solve two-variable linear equations, functions, and inequalities
	        ▪ interpret the solution to identify the number of acceptable solutions (e.g., zero, one, infinitely many solutions)
	        ▪ solve, graph, and check the solution to two-variable linear equations and inequalities including absolute value
	        ▪ understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously
	        ▪ solve, graph, and check the solution to two-variable systems of linear equations and inequalities using:
	            • substitution
	            • graphing

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<td>Problem Solving

Statistics and Probability</td>
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- linear combination (elimination)
- write the equation of a line using:
  - data table
  - linear graph
  - point-slope form
  - slope-intercept form
  - standard form
  - slope formula
  - x-intercept and y-intercept
  - parallel and perpendicular slopes
- construct a viable argument to justify a solution method

- solve quadratic equations
  - understand that solutions to a quadratic equation correspond to the x-intercepts of their graphs
  - interpret the solution to identify the number of acceptable solutions (e.g., zero, one, and two)
  - solve and check the solution to any quadratic equation and inequality using:
    - graphing – intercepts, vertex, maxima, minima, and line of symmetry
    - quadratic formula: x = [-b +/- (b2 – 4ac)1/2]/2°
    - factoring
      1. factor polynomials using the Distributive Property (GCF)
      2. factor trinomials and perfect square trinomials
    - formula for the line of symmetry: x = -b/2a
    - completing the square
    - standard graphing form: y = a(x-b)2 + c
    - standard form: y = ax2 + bx + c
  - construct a viable argument to justify a solution method
  - write a quadratic equation given a graph of a parabola or set of values
- radical equations
  - interpret the solution to identify the number of acceptable solutions (e.g., extraneous solutions)
  - solve and check the solution to radical equations by:
    - completing the square
    - squaring both sides of the equation
    - applying Pythagorean Theorem
  - construct a viable argument to justify a solution method

- rational equations
  - interpret the solution to identify the number of acceptable solutions (e.g., extraneous solutions)
  - solve and check the solution to rational equations using the concepts of:
    - the conjugate
    - least common denominator
    - cross-multiplication
  - construct a viable argument to justify a solution method

- create equations and inequalities in one or two variables and use them to solve problems
- solve standard word problems using one or two variables including:
  - uniform motion or distance
  - consecutive integers
  - geometric properties of perimeter, area, distance, and Pythagorean Theorem
  - mixture or solution
  - work
  - combination
  - place value or digit

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	(continued) - age - scientific notation - exponential growth and decay - indirect variation - interpret the solution to identify the number of acceptable solutions (e.g., extraneous solutions) - evaluate solutions for reasonableness, accuracy, and completeness - Identify and interpret data with exponential behavior. - investigate patterns of association in two-variable data - construct and interpret scatter plots to investigate patterns of association such as positive and negative correlation, linear and nonlinear associations, and outliers - Write equations of lines of best fit.

Model Traditional Pathway: High School Geometry

Introduction

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course.

The high school Geometry standards, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. Each high school should design their course syllabus using the following standards. For this high school Geometry course, instructional time should focus on the following six critical area units:

Unit1 Congruence, Proof, and Constructions: *In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work.*

Unit 2 Similarity, Proof, and Trigonometry: *Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles, building on students' work with quadratic equations. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles.*

Unit 3 Extending to Three Dimensions*: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.*

Unit 4 Connecting Algebra and Geometry through Coordinates: *Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, which relates back to work done in the first course. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.*

Unit 5 Circles With and Without Coordinates: *In this unit students prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations, which relates back to work done in the first course, to determine intersections between lines and circles or parabolas and between two circles.*

Unit 6 Applications of Probability: *Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.*

GEOMETRY

Unit 1: Congruence, Proof, and Constructions

Students will:

- Experiment with transformations in the plane.
- Understand congruence in terms of rigid motions.
- Prove geometric theorems.
- Make geometric constructions.

Unit 2: Similarity, Proof, and Trigonometry

Students will:

- Understand similarity in terms of similarity transformations.
- Prove theorems involving similarity.
- Define trigonometric ratios and solve problems involving right triangles.
- Apply geometric concepts in modeling situations.
- Apply trigonometry to general triangles.

Unit 3: Extending to Three Dimensions

Students will:

- Explain volume formulas and use them to solve problems.
- Visualize the relation between two-dimensional and three-dimensional objects.
- Apply geometric concepts in modeling situations.

Unit 4: Connecting Algebra and Geometry through Coordinates

Students will:

- Use coordinates to prove simple geometric theorems algebraically.
- Translate between the geometric description and the equation for a conic section.

Unit 5: Circles With and Without Coordinates

Students will:

- Understand and apply theorems about circles.
- Find arc lengths and areas of sectors of circles.
- Translate between the geometric description and the equation for a conic section.
- Use coordinates to prove simple geometric theorem algebraically.
- Apply geometric concepts in modeling situations.

Unit 6: Applications of Probability

Students will:

- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.
- Use probability to evaluate outcomes of decisions.

Unit 1: Congruence, Proof and Constructions

- Experiment with transformations in the plane.

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs.
3. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
4. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
5. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
6. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

- Understand congruence in terms of rigid motions.

7. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
8. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
9. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

- Prove geometric theorems.

10. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
11. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
12. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

- Make geometric constructions.

13. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
14. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Unit 2: Similarity, Proof, and Trigonometry

- Understand Similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor.
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

- Prove theorems involving similarity.

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

- Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

- Apply geometric concepts in modeling situations.

9. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
10. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
11. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
12. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. *

- Apply trigonometry to general triangles.

13. Derive the formula *A* = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. +
14. Prove the Laws of Sines and Cosines and use them to solve problems. +
15. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles(e.g., surveying problems, resultant forces). +

Unit 3: Extending to Three Dimensions

- Explain volume formulas and use them to solve problems.

    1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
    2. Give an informal argument using Cavalierir's principle for the formulas for the volume of a sphere and other solid figures.
    3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. *

- Visualize the relation between two dimensional and three-dimensional objects.

    4. Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

- Apply geometric concepts in modeling situations.

5. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

6. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★
7. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★
8. MA.4.Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. ★

- Use coordinates to prove simple geometric theorems algebraically.

9. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
10. Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
11. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
12. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

- Translate between the geometric description and the equation for a conic section.

13. Derive the equation of a parabola given a focus and directrix.
14. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
15. Derive the equation of a parabola given a focus and directrix.

Unit 4: Connecting Algebra and Geometry Through Coordinates

- Use coordinates to prove simple geometric theorems algebraically.

1. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
2. Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
3. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
4. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

- Translate between the geometric description and the equation for a conic section.

5. Derive the equation of a parabola given a focus and directrix.

Unit 5: Circles With and Without Coordinates

- Understand and apply theorems about circles.

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. Construct a tangent line from a point outside a given circle to the circle. +

- Find arc lengths and areas of sectors of circles.

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

- Translate between the geometric description and the equation for a conic section.

6. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
7. Derive the equation of a parabola given a focus and directrix

- Use coordinates to prove simple geometric theorems algebraically.

8. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).
9. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
10. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
11. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

- Apply geometric concepts in modeling situations.

12. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★
13. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★
14. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★
15. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. ★

★ indicates Modeling standard.
(+) indicates standard beyond College and Career Ready.

Unit 6: Applications of Probability

- Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. ★
7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model. ★
8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. ★
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. ★

- Use probability to evaluate outcomes of decisions.

10. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). +
11. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). +
12. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ★ +

Model Traditional Pathway: High School Algebra II

Introduction

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms.

The standards for this Algebra II course[2], together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. Each high school should design their course syllabus using the following standards. For this high school Algebra II course, instructional time should focus on the following six critical area units:

***Unit 1 Polynomial, Rational, and Radical Relationships:** This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.*

***Unit 2 Trigonometric Functions**: Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.*

***Unit 3 Modeling with Functions:** In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.*

[2] Adopted from the Archdiocese of Hartford Mathematics Standards, the Common Core State Standards for Mathematics and Appendix A: *Designing High School Courses based on the Common Core State Standards for Mathematics*, 2010.

Unit 4 ***Inferences and Conclusions from Data****: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn. In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.*

ALGEBRA II

Unit 1: Polynomial, Rational, and Radical Relationships

Students will:

- Perform arithmetic operations with complex numbers.
- Use complex numbers in polynomial identities and equations.
- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.
- Perform arithmetic operations on polynomials.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.
- Understand solving equations as a process of reasoning and explain the reasoning.
- Represent and solve equations and inequalities graphically.
- Analyze functions using different representations.

Unit 2: Trigonometric Functions

Students will:

- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric function.
- Prove and apply trigonometric identities.

Unit 3: Modeling with Functions

Students will:

- Create equations that describe numbers or relationships.
- Interpret functions that arise in applications in terms of a context.
- Analyze functions using different representations.
- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.
- Construct and compare linear, quadratic, and exponential models and solve problems.

Unit 4: Inferences and Conclusions from Data

Students will:

- Summarize, represent, and interpret data on single count or measurement variable.
- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments and observational studies.
- Use probability to evaluate outcomes of decisions

Algebra II

Unit 1: Polynomial, Rational, and Radical Relationships

- Perform arithmetic operations with complex numbers.

1. Know there is a complex number *i* such that *i*2 = −1, and every complex number has the form *a + bi* with *a* and *b* real.
2. Use the relation *i*2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

- Use complex numbers in polynomial identities and equations.

3. Solve quadratic equations with real coefficients that have complex solutions.
4. Extend polynomial identities to the complex numbers. *For example, rewrite x2 + 4 as (x + 2i)(x – 2i). +*
5. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. +

- Interpret the structure of expressions. *Extend to polynomial and rational expressions.*

6. Interpret expressions that represent a quantity in terms of its context.★
7. Interpret parts of an expression, such as terms, factors, and coefficients.
8. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n as the product of P and a factor not depending on P.*
9. Use the structure of an expression to identify ways to rewrite it. *For example, see x4 – y4 as (x2)2 – (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 – y2)(x2 + y2).*

- Write expressions in equivalent forms to solve problems.

10. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★

- Perform arithmetic operations on polynomials.

11. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- Understand the relationship between zeros and factors of polynomials.

12. Know and apply the Remainder Theorem: For a polynomial *p(x)* and a number *a*, the remainder on division by *x – a* is *p(a)*, so *p(a)* = 0 if and only *if (x – a)* is a factor of p(x).
13. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

- Use polynomial identities to solve problems.

14. Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity *(x2 + y2)2 = (x2 – y2)2 + (2xy)2* can be used to generate Pythagorean triples.
15. Know and apply the Binomial Theorem for the expansion of *(x + y)n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. +

- Rewrite rational expressions

16. Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.
17. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. +

- Understand solving equations as a process of reasoning and explain the reasoning.

18. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

- Represent and solve equations and inequalities graphically.

19. Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

- Analyze functions using different representations.

20. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
    - Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

Unit 2: Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle.

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

- Model periodic phenomena with trigonometric functions.

3. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

- Prove and apply trigonometric identities.

4. Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.

Unit 3: Modeling with Functions

- Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

- Interpret functions that arise in applications in terms of a context.

5. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
6. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
7. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

- Analyze functions using different representations.

8. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
    - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

9. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
10. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

- Build a function that models a relationship between two quantities.

11. Write a function that describes a relationship between two quantities.*
    b. Combine standard function types using arithmetic operations.
    For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model..

- Build new functions from existing functions.

12. Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
13. Find inverse functions.
    a. Solve an equation of the form *f(x) = c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) = 2 x3 or f(x) = (x+1)/(x-1) for x ≠ 1.*

- Construct and compare linear, quadratic, and exponential models and solve problems.

14. For exponential models, express as a logarithm the solution to a *bct = d* where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Unit 4: Inferences and Conclusions from Data

- Summarize, represent, and interpret data on a single count or measurement variable.

1. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

- Understand and evaluate random processes underlying statistical experiments.

2. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
3. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

4. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
5. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
6. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
7. Evaluate reports based on data.

- Use probability to evaluate outcomes of decisions.

8. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). +
9. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). +

Model Advanced Course: Model Precalculus

Introduction

Precalculus combines the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus, and strengthens students' conceptual understanding of problems and mathematical reasoning in solving problems. Facility with these topics is especially important for students intending to study calculus, physics, and other sciences, and/or engineering in college. Because the standards for this course are (+) standards, students selecting this Model Precalculus[3] course should have met the college and career ready standards.

Each high school should design their course syllabus using the following standards. For the high school Model Precalculus course, instructional time should focus on the following four critical area units: (1) extend work with complex numbers; (2) expand understanding of logarithms and exponential functions; (3) use characteristics of polynomial and rational functions to sketch graphs of those functions; and (4) perform operations with vectors.

Unit 1 ***Extend Work with Complex Numbers*****:** Students continue their work with complex numbers. They perform arithmetic operations with complex numbers and represent them and the operations on the complex plane. Students investigate and identify the characteristics of the graphs of polar equations, using graphing tools. This includes classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

Unit 2 ***Expand Understanding of Logarithms and Exponential Functions*****:** Students expand their understanding of functions to include logarithmic and trigonometric functions. They investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. This includes the role of *e*, natural and common logarithms, laws of exponents and logarithms, and the solutions of logarithmic and exponential equations. Students model periodic phenomena with trigonometric functions and prove trigonometric identities. Other trigonometric topics include reviewing unit circle trigonometry, proving trigonometric identities, solving trigonometric equations, and graphing trigonometric functions.

Unit 3 *Use Characteristics of Polynomial and Rational Functions to Sketch Graphs of Functions***:** Students investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. They determine zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Students translate between the geometric description and equation of conic sections. They deepen their understanding of the Fundamental Theorem of Algebra.

Unit 4 *Perform Operations with Vectors***:** Students perform operations with vectors in the coordinate plane and solve practical problems using vectors. This includes the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

[3] Adopted from the Massachusetts State Curriculum, the Common Core State Standards for Mathematics, and Appendix A: *Designing High School Courses based on the Common Core State Standards for Mathematics*, 2010

Precalculus

Number and Quantity

The Complex Number System

- **Perform arithmetic operations with complex numbers.**

1. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

- **Represent complex numbers and their operations on the complex plane.**

2. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
3. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $\left(-1+\sqrt{3i}\right)^3=8$ *because* $\left(-1+\sqrt{3i}\right)$ *has modulus 2 and argument 120°.*
4. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

- **Use complex numbers in polynomial identities and equations.**

5. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.
6. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities

- **Represent and model with vector quantities.**

7. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).
8. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
9. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

(+) indicates standard beyond College and Career Ready.

- **Perform operations on vectors.**

10. (+) Add and subtract vectors.
    a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
    b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
    c. (+) Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

11. (+) Multiply a vector by a scalar.
    a. (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
    b. (+) Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

- **Perform operations on matrices and use matrices in applications.**

12. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
13. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
14. (+) Add, subtract, and multiply matrices of appropriate dimensions.
15. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
16. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
17. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
18. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Algebra

Arithmetic with Polynomials and Rational Expressions

- **Use polynomial identities to solve problems.**

19. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[4]

- **Rewrite rational expressions.**

20. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
21. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Reasoning with Equations and Inequalities

- **Solve systems of equations.**

22. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
23. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Functions

Interpreting Functions

- **Analyze functions using different representations.**

24. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ⋆
    c. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ★

Building Functions

- **Build a function that models a relationship between two quantities.**

25. Write a function that describes a relationship between two quantities. ⋆

[4] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

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d. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* ★

- **Build new functions from existing functions.**

26. Find inverse functions.
27. b. (+) Verify by composition that one function is the inverse of another.
    d. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    e. (+) Produce an invertible function from a non-invertible function by restricting the domain.
28. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Trigonometric Functions

- **Extend the domain of trigonometric functions using the unit circle.**

29. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
30. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

- **Model periodic phenomena with trigonometric functions.**

31. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
32. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★

- **Prove and apply trigonometric identities.**

33. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

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Geometry

Similarity, Right Triangles, and Trigonometry

- **Apply trigonometry to general triangles.**

34. (+) Derive the formula $A = ½ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
35. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
36. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles

- **Understand and apply theorems about circles.**

37. Construct a tangent line from a point outside a given circle to the circle.

Expressing Geometric Properties with Equations

- **Translate between the geometric description and the equation for a conic section.**

38. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
39. MA.3.a. (+) Use equations and graphs of conic sections to model real-world problems. ★

Geometric Measurement and Dimension

- **Explain volume formulas and use them to solve problems.**

40. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

- **Visualize relationships between two-dimensional and three-dimensional objects.**

41. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

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